

Displacement Capacity of Lightly Reinforced Rectangular Concrete Walls

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ABSTRACT:

This paper investigates the displacement capacity of lightly reinforced rectangular walls. The design parameters and detailing of the walls considered in this study are typical of low-to-moderate seismic regions such as Australia. Obtaining accurate estimates of the displacement capacity of lightly reinforced walls is a vital step towards producing realistic estimates of fragility curves of RC buildings in low-to-moderate seismic regions. It is shown in this investigation that the nominal yield displacement capacity for reinforced concrete walls that have low amounts of longitudinal reinforcement can sometimes be overestimated using the existing equations. An alternative approach is proposed here which gives a better match with experimental and numerical results and is particularly important for the reinforced concrete walls that exhibit a single crack in the plastic hinge zone. An ultimate curvature equation is proposed for when a wall is estimated to form a single crack in the plastic hinge region. A plastic hinge length equation which has been specifically derived for these types of walls is also introduced to estimate the plastic displacement of the wall due to plastic rotation at the base. Experimental and finite element modelling results are used to compare the displacement capacity predictions.

Keywords: Non-ductile, single crack, strain penetration, low reinforcement

1. Introduction

For seismic design and assessment purposes it is typical to use a bilinear (or trilinear) approximation of the force-displacement response of reinforced concrete walls (Priestley *et al.*, 2007; Wibowo *et al.*, 2013), as shown in Figure 1. The point of first yield (Δ'_y) is defined as that at which the tension reinforcement at the outermost position on the tensile side first reaches the yield strain. The line that extends from this point to the nominal yield (Δ_y) point is calculated using Equation 1. The ultimate displacement (Δ_u) is the summation of the nominal yield displacement and the plastic displacement (Δ_p).

$$\Delta_y = \frac{\Delta'_y F_u}{F_y} \quad (1)$$

where F_y and F_u are the force at first yield and ultimate force respectively.

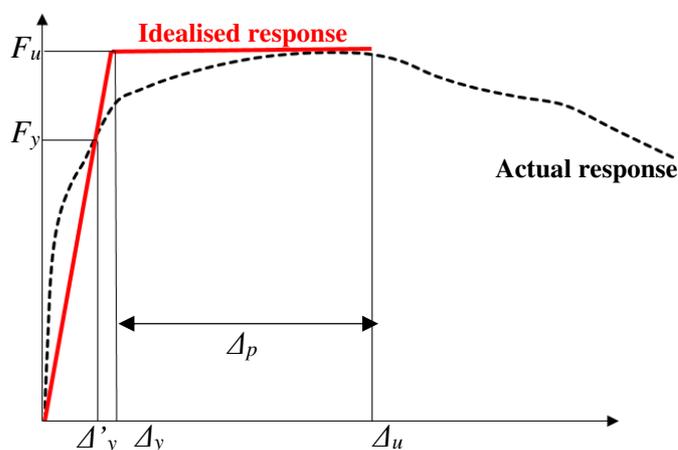


Figure 1 Idealised Moment-Curvature with nominal yield displacement

These key displacement points in Figure 1 can commonly be calculated from a moment-curvature analysis performed for the reinforced concrete (RC) section, or alternatively can be estimated using simplified equations. For example, Priestley *et al.* (2007) recommends that Equation 2 be used to calculate the nominal yield displacement of an RC wall, where the approximate nominal yield curvature (Φ_y) is estimated using Equation 3.

$$\Delta_y = \frac{\Phi_y H_e^2}{2} \left(1 - \frac{H_e}{3H_n}\right) \quad (2)$$

where H_e is the effective height of the wall and H_n is the total height of the wall.

$$\Phi_y = \frac{2\varepsilon_{sy}}{L_w} \quad (3)$$

where ε_{sy} is the yield strain of the reinforcing steel and L_w is the wall length.

However, these equations have been derived for ‘typical conditions’, where a linear curvature profile is assumed for the wall reaching the yield strain (and curvature) at the base. This assumption may not be applicable to lightly reinforced walls that exhibit a single-crack failure at the base of the wall. In this case, the wall acts as a rigid body and must be separated from the flexural deformation that is classically obtained from the curvature distribution over the height of the wall. This is discussed by Wibowo *et al.* (2016) for lightly RC columns, where the fixed end assumption, which is assumed in the traditional moment-curvature analysis, ‘ignores the yield penetration effect associated with reinforcement elongation and bond slip in the anchorage zone’. This can also be applied to single-crack failures in lightly reinforced walls, where yield penetration and bond slip are the primary contributors to the overall global displacement of the wall.

Single-crack-failures in lightly reinforced concrete (RC) walls have been observed in the recent Canterbury earthquake sequence in 2011. For example, an RC wall in the Gallery Apartments exhibited a single crack at the base of the wall. On further inspection, longitudinal reinforcing bars that crossed this crack were found to have fractured (Henry, 2013; Sritharan *et al.*, 2014). ‘The building’s [Gallery Apartments] overall damage state may be described as being at near collapse. A potentially catastrophic failure might have been observed for a slightly longer duration of severe ground shaking’ (Morris *et al.*, 2015). In past experimental testing of RC wall elements, it has been common for researchers to observe the development of a plastic hinge zone at the base of the wall in which cracks are well distributed. The reason that single crack failures have not usually been observed in these tests is because of the high amounts of longitudinal reinforcement used in these specimens. This would be common for RC wall designs in high seismic regions, but not in low-to-moderate seismic regions, such as Australia, where the majority of the building stock includes RC wall elements that have longitudinal reinforcement ratios (ρ_{wv}) less than 1% (Albidah *et al.*, 2013; Wibowo *et al.*, 2013).

The aim of this paper is to investigate the (nominal) yield and plastic displacement capacities of RC walls. An equation suitable for predicting the minimum longitudinal reinforcement required for secondary cracking is introduced. Coefficients have then been derived from finite element (FE) modelling results to be used in expressions to improve the estimates of the nominal yield displacement of lightly RC walls. Further to this, in order to estimate the contribution to the ultimate displacement capacity from the displacements due to plastic rotation at the base, a plastic hinge length equation, specifically derived for these types of walls, is introduced. Moreover, an equation is proposed to estimate the average curvature at the single crack at the base when the ultimate displacement capacity is reached. The displacement estimates based on the proposed equations are substantiated by comparisons to the experimental and FE results. Achieving an accurate estimation of the displacement capacity of an RC wall is crucial in obtaining realistic results when constructing fragility curves. The fragility curves consider many cases and it would be very time-consuming to base them entirely on FE results; it is more expedient to rely upon reasonable estimates made with formulae that match well with the FE results within the range of interest.

2. Minimum longitudinal reinforcement required

The authors have recently conducted an extensive numerical study of lightly reinforced and unconfined walls (Hoult *et al.*, 2016b), detailed in accordance with AS3600:2009 (Standards Australia, 2009). The results from the study indicated that a simple Secondary Cracking Model (SCM) can successfully estimate the minimum longitudinal reinforcement ratio ($\rho_{wv.min}$) required to initiate secondary cracking in an RC wall. Using the SCM, Equation 4 has been derived to estimate the minimum longitudinal reinforcement ratio ($\rho_{wv.min}$).

$$\rho_{wv.min} = \frac{(t_w - n_t d_{bt}) f_{ct.fl}}{f_u t_w} \quad (4)$$

where t_w is the thickness of the wall, n_t is the number of layers of transverse reinforcing bars, d_{bt} is the diameter of transverse reinforcing bars, $f_{ct.fl}$ is the flexural tensile strength of the concrete and f_u is the ultimate strength of the longitudinal reinforcing bars.

3. Yield displacement as a function of ρ_{wv}

Several lightly reinforced walls were analysed to obtain the displacement at first yield (Δ'_y). VecTor2 (Wong *et al.*, 2013) is a state-of-the-art finite element (FE) modelling program for plane RC sections that is based on the disturbed stress field model (Vecchio *et al.*, 2000). The program was validated by comparing the prediction of the force-displacement behaviour (cyclic and monotonic) and strain distributions for some lightly reinforced RC walls (Hoult *et al.*, 2016a, 2016b). The constitutive and materials models used in VecTor2 include the Popovics normal-strength concrete compression model, which was adopted from the recommendations from Palermo and Vecchio (2007). The model for the reinforcing steel is represented by the stress-strain curve as suggested in Seckin (1981). Recent investigations by the authors using VecTor2 have shown that the Kupfer/Richart model (Kupfer *et al.*, 1969; Richart *et al.*, 1928) gives an overall better result for walls that are governed by compression. It was assumed that the reinforcement had perfect bond, an approach that has also been adopted in Lu *et al.* (2015). More details of the extensive list of models chosen for the VecTor2 analyses can be found in Hoult *et al.* (2016a) and Hoult *et al.* (2016b). For all of the walls modelled here, a mesh size of approximately $0.5t_w$ to $1.0t_w$ was used, where the 3:2 aspect ratio limit was recommended in Wong and Vecchio (2002). The axial load was distributed and applied to all nodes at the top of the wall (held constant throughout the analysis), while the same nodes were subjected to a lateral displacement for the monotonic nonlinear pushover analyses.

The two walls chosen for the initial study have wall lengths (L_w) of 3 and 6 metres and are 200 mm thick, with effective heights (H_e) corresponding to 7.35m and 35 m respectively. The axial load ratio (ALR) for both walls was 5%. The mean in situ compressive strength of concrete (f_{cmi}) is 40 MPa and the mean mechanical properties of steel from Menegon *et al.* (2015b) for D500N bars were used, which corresponds to a ε_{sy} of 0.0027 (assuming Young's Modulus of the reinforcing steel is 200 GPa). The nominal yield displacement results from VecTor2 as a function of the longitudinal reinforcement ratio (ρ_{wv}) are illustrated in Figure 2. In this case, the nominal yield displacement (Δ'_y) was calculated using Equation 1, by extracting the values for Δ'_y , F_u and F_y from the FE results for each increment of ρ_{wv} . The results from this study

indicate that the nominal yield displacement increases as a function of the longitudinal reinforcement ratio (ρ_{wv}) of the wall. This is discussed by Wallace and Moehle (1992) for the nominal yield curvature (Φ_y), which ‘increases with increasing steel ratio and axial load’. Moreover, the estimation from Priestley *et al.* (2007) using Equations 2 and 3 vastly overestimates the nominal yield displacement over the entire ρ_{wv} range. As discussed in Priestley and Kowalski (1998), the relationship for yield curvature (given in Equation 3) ‘should not be extrapolated beyond the range $300 \leq f_y \leq 500$ MPa nor beyond the limits of longitudinal steel ratio’, which were between $0.01 \leq \rho_{wv} \leq 0.04$. Given that the lower characteristic value of the yield stress (f_y) of D500N reinforcing bars used in Australia is 500 MPa (Standards Australia/New Zealand, 2001) the actual value of f_y will typically be higher than 500 MPa. Moreover, the majority of RC walls found in this region have longitudinal reinforcement ratios lower than 0.01 (Albidah *et al.*, 2013; Wibowo *et al.*, 2013). Hence the equation for estimating the nominal yield curvature for rectangular walls in Australia needs further investigation.

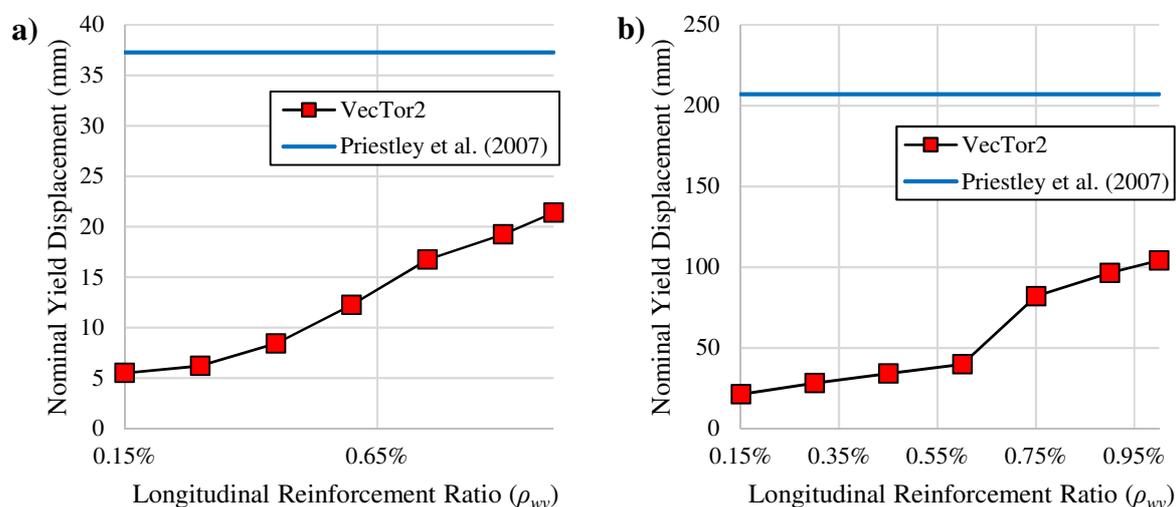


Figure 2 Nominal yield displacement as a function of longitudinal reinforcement for (a) 3 metre wall and (b) 6 metre wall

It is also interesting to note that the resulting nominal yield displacement from VecTor2 is also significantly lower than the estimate from Priestley *et al.* (2007) for walls with ρ_{wv} larger than $\rho_{wv.min}$. It has been shown that for certain walls with well distributed cracking a linear curvature profile at yield is able to provide a good approximation to the yield displacement that corresponds to a more realistic curvature (Priestley *et al.*, 2007). For example, Equation 2 has been derived by assuming a linear curvature profile, which has been shown to give only a slightly higher yield displacement result in comparison to a more ‘realistic’ curvature profile of a wall which has an uncracked upper portion (50% of wall height) if ignoring tension shift effects (Priestley *et al.*, 2007). However, this same approach is not likely to give a close estimate of the yield displacement for walls with a low amount of longitudinal reinforcement (and unconfined boundary ends), where flexural cracking above the base of the wall does not extend to 50% of the wall height.

To investigate this, while highlighting the difference in behaviour for walls with a single crack and those in which well distributed cracking is observed, values of ρ_{wv} of 0.15% and 1.00% have been taken and are used to compare key strain and curvature distributions. Using Equation 4, it is estimated that both walls need a ρ_{wv} of approximately 0.50% to allow secondary cracking. This gives an insight into the reasons for the differences in estimates of yield displacement that have been discussed above. Figure 3(a) shows the results for the 3 metre walls with a ρ_{wv} of 0.15% and 1.00% respectively at the point at which the yield strain, ϵ_{sy} , has just been reached in the outermost layer of tensile reinforcement at the base of the wall. For the wall with a single crack ($\rho_{wv} = 0.15\%$), the strain distribution at yield is clearly concentrated at the base of the wall and the tensile strain rapidly diminishes over a small length up the height of the wall. This is in contrast to the wall with a 1.00% longitudinal reinforcement ratio, where the strains appear to be more distributed up the wall height. Figure 3(b) shows the curvature distribution results for the walls with secondary cracking (ρ_{wv} of 0.60%, 0.75% and 1.00%) over the entire wall height (of 7.35 m). Superimposed on this figure is a linear curvature profile. Although it appears some yielding (and thus, some cracking) have occurred in the majority of the lower 50% of the wall height, the actual curvature profiles are far different from the approximate linear curvature profile. Moreover, the curvature distribution becomes more linear as the longitudinal reinforcement ratio increases, which explains the trend illustrated in Figure 2.

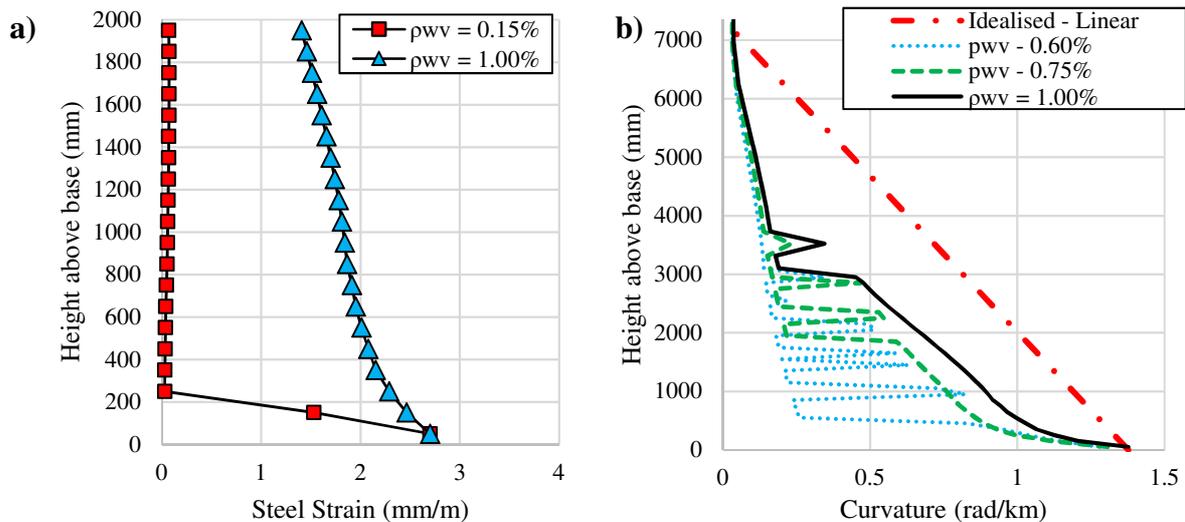


Figure 3 (a) Steel strain distribution and (b) curvature distribution for the 3 metre wall in VecTor2

The results here have shown that a review is needed of the displacement capacity equations for the lightly reinforced walls typically found in regions of low-to-moderate seismicity. In the next section coefficients are derived that can be used in equations to better predict the nominal yield displacement of lightly reinforced walls.

4. Nominal Yield Displacement Capacity

For cantilever wall structures, the nominal yield displacement can be predicted by using the expression in the form of Equation 5 (Tjhin *et al.*, 2004). The yield displacement coefficient (k_d) depends on the curvature distribution up the height of the wall. As previously discussed

in Sections 1 and 3, lightly reinforced concrete walls that exhibit a single crack will have a concentration of strain (and therefore curvature) over a small, localised length of the wall height from the base. Hence, estimating the Δ_y with Equation 5 using k_Δ is inappropriate for these walls that are likely to form a single crack. Instead, the yield displacement can be estimated using Equations 6 and 7, which have been derived from a study focusing on the experimental testing of lightly reinforced concrete elements (Altheeb, 2016). The $\Delta_{sp.e}$ is the deformation due to elastic strain penetration.

$$\Delta_y = k_\Delta \phi_y H_e^2 \quad (5)$$

$$\Delta_y = \Delta_{sp.e} = \frac{\Delta_{slip.elastic}}{0.5L_w - d_c} H_e \quad (6)$$

where d_c is the cover distance of the longitudinal reinforcing bars.

$$\Delta_{slip.elastic} = \frac{\varepsilon_{sy} f_{sy} d_{bl}}{1.2(d_{bl})^{0.5} \sqrt{f'_c}} \quad (7)$$

where f_{sy} and d_{bl} is the yield strength and diameter of the longitudinal reinforcing bars.

For walls that do form secondary cracking, the nominal yield curvature (Φ_y) can be estimated using an expression in the form of Equation 8, where k_ϕ is the nominal yield curvature coefficient.

$$\phi_y = \frac{k_\phi}{L_w} \quad (8)$$

The k_ϕ value is also commonly dependent on the yield strain value of the reinforcing bars used; e.g. $k_\phi = 2\varepsilon_{sy}$ from Priestley *et al.* (2007) for rectangular concrete walls. FE results for walls that exhibited secondary cracking will be used to compare and derive (if necessary) k_Δ and k_ϕ values for the lightly reinforced, unconfined walls.

The FE modelling program VecTor2 was used to conduct further nonlinear pushover analyses using the constitutive models and mesh guidelines discussed in Hoult *et al.* (2016b) and Section 3. The sensitivity of the coefficients to several different parameters was investigated; these parameters were the wall length (L_w), axial load ratio (ALR) and longitudinal reinforcement ratio (ρ_{wy}). A realistic range of values was taken for each parameter, given in Table 1. Mean values of the mechanical properties were taken from Menegon *et al.* (2015b) for the D500N steel reinforcing bars that are commonly used in Australia. The f_{cmi} was held constant at 40MPa, while the $ALRs$ considered were 1.5% and 10%, a lower and upper limit found to be within the recommended and observed values from Henry (2013) and Albidah *et al.* (2013). All of the walls had an aspect ratio ($a = H_e/L_w$) higher than 3 so that flexural behaviour would govern; the effective height that correspond to the 3, 6 and 9 metre walls are 12.25, 19.6 and

34.3 metres. Importantly, the ρ_{wv} considered for this study were chosen to be higher than the $\rho_{wv.min}$ calculated using Equation 1 (approximately 0.50%). This is because the coefficients and corresponding equations will be used for walls with flexural behaviour; if ρ_{wv} is less than $\rho_{wv.min}$ other equations based on strain penetration behaviour are needed as discussed previously.

Table 1 Values used for several parameters of the RC walls investigated

L_w (m)	ALR (%)	ρ_{wv} (%)
3, 6, 9	2.5, 10	0.6, 0.75, 0.9

Using the VecTor2 results, the yield displacement (Δ'_y) was determined as the displacement at which the outermost longitudinal reinforcing bar reached or exceeded the yield strain ϵ_{sy} (approximately 0.0027). The curvature at first yield (Φ'_y) was also calculated from the steel and concrete strains reached in the respective extreme fibre edges at the yield displacement. The nominal yield curvature (Φ_y) was thus calculated using Equation 9 with the corresponding moment at first yield (M_y) and ultimate moment (M_u). Similarly, the nominal yield displacement (Δ_y) was calculated using Equation 1. The nominal yield curvature coefficient (k_ϕ) and yield displacement coefficient (k_Δ) were found by using the curvature and displacement results from VecTor2 and rearranging Equations 8 and 5 respectively. The results from VecTor2 and the corresponding coefficients are given in Table C1 (Appendix C).

$$\phi_y = \frac{\phi'_y M_u}{M_y} \quad (9)$$

Table C1 shows that the commonly used expression from Equation 3 overestimates the nominal yield curvature in comparison to the VecTor2 results. Instead, a k_ϕ value of $1.6\epsilon_{sy}$ was found to be more applicable, which has been derived from the minimum of the sum of the squares.

The k_Δ results in Table C1 indicate some variability, with a high dependence on the amount of longitudinal reinforcement in the wall, as was found in Section 3. Therefore, Equation 10 can be used to estimate k_Δ based on the ρ_{wv} of the wall. A minimum and maximum value of 0.08 and 0.24 for k_Δ respectively were chosen based on the results of this study

$$0.08 \leq k_\Delta = 39\rho_{wv} - 0.12 \leq 0.24 \quad (10)$$

5. Plastic Displacement Capacity

As previously discussed in Section 1, observations from the Christchurch earthquake indicated that some lightly reinforced walls may fail in a non-ductile fashion with a single crack in the plastic hinge zone. In this case, the curvature is concentrated over a small height at the base of the wall. Furthermore, a typical moment-curvature analysis may not be able to predict the ultimate curvature achieved by these types of walls as the performance is governed by strain penetration deformation rather than flexural deformation, where the assumptions inherently built in to a moment-curvature analyses no longer apply. Research by Wibowo *et al.* (2016) has led to an alternative method for calculating the moment-curvature for lightly reinforced concrete columns. Moreover, Wibowo *et al.* (2016) offer a simplified alternative for the

moment and curvature. However, these equations may not be applicable to RC walls since there is typically a greater axial load ratio (ALR) and longitudinal reinforcement ratio (ρ_{wv}) in lightly reinforced columns in comparison to lightly reinforced walls. Instead, the proposed estimation here stems from treating the plastic behaviour due to yield penetration and bond slip at the base as a plastic rotation due to an estimated curvature multiplied by a plastic hinge length. This estimate of the plastic curvature (Φ_p) of the walls with a single-crack failure is illustrated in Figure 4. The Φ_p can be calculated using Equation 11, which conservatively assumes that no compression region exists over the length of the wall. An ultimate strain value equal to 0.6 of the uniform elongation strain (ε_{su}) for the reinforcing steel is used due to the recommendations by Priestley *et al.* (2007) for low-cycle fatigue. The yield strain (ε_{sy}) value is subtracted from the ultimate strain ($0.6\varepsilon_{su}$). In contrast, for walls that achieve secondary cracking, and hence well distributed cracks within the plastic hinge zone, a typical moment-curvature analysis can be used to obtain an estimate of the ultimate curvature (Φ_u), as it is assumed that the walls' performance will be governed by flexural behaviour. In this case Φ_p can then be calculated by subtracting the estimated nominal yield curvature (e.g. $\Phi_p = \Phi_u - \Phi_y$).

$$\phi_p = \frac{0.6\varepsilon_{su} - \varepsilon_{sy}}{L_w} \quad (11)$$

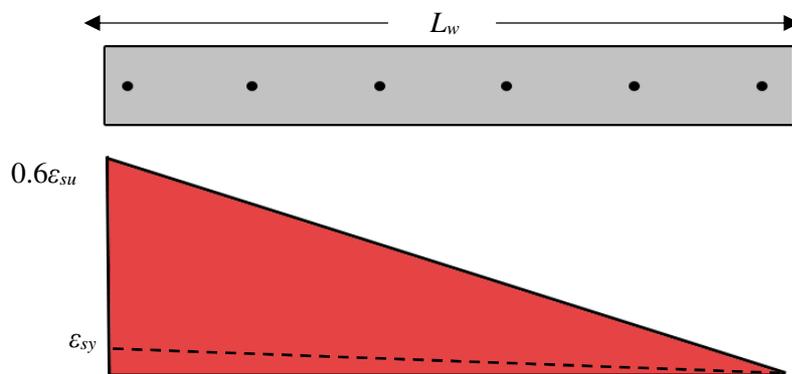


Figure 4 Idealised strain reached at the ultimate state for a single-crack-failure wall

The plastic displacement is then calculated using Equation 12. For walls that form secondary cracks, the plastic hinge length (L_p) can be calculated for lightly reinforced and unconfined concrete rectangular walls using Equation 13. This equation has been derived from regression analysis using results of an extensive study testing the effects of numerous parameters by the authors for these types of walls (Hoult *et al.*, 2016b). It is limited to walls with $ALRs$ less than 10%. Hoult *et al.* (2016b) recommend that a more conservative equation be used for design purposes. Moreover, the plastic strain penetration length (L_{sp}) was not included in the original study; thus, the L_{sp} estimated by Altheeb (2016) (Equation 14) has been included into Equation 13 to give a more realistic length for assessment purposes.

$$\Delta_p = (\phi_p)L_p H_e \quad (12)$$

$$L_p = (0.10L_w + 0.075H_e)(1 - 6ALR) + L_{sp} \leq 0.5L_w \quad (13)$$

$$L_{sp} = \frac{(f_{su} - f_{sy})d_{bl}^{1.2}}{4\sqrt{f'_c}} \quad (14)$$

If a wall is estimated to have a single, primary crack, the plasticity is confined to a small length of the wall. The authors previously suggested that a value of 100 mm could be used here (Hoult *et al.*, 2016b); by using the product of this length and the plastic curvature estimated in Equation 11 a conservative estimate of the plastic rotation could be obtained that incorporates strain penetration on each side of the crack. A less conservative value of 150 mm is used here for assessment purposes, which corresponds to the mean plastic hinge length for the walls assessed in Hoult *et al.* (2016b).

6. Comparison with experimental and FEM walls

Recent experimental testing on lightly reinforced rectangular concrete walls have been conducted in Australia using D500N type reinforcing steel (Albidah, 2016; Altheeb, 2016). One of the two walls tested and reported here formed a single-crack at the base of the wall, which had a ρ_{wv} content less than the $\rho_{wv.min}$ estimated using Equation 4. This presents an opportunity to test the proposed equations against the experimental data. For the sake of brevity, the reader is referred to the corresponding reports for full details of the wall specimens (Albidah, 2016; Altheeb, 2016). VecTor2 provided good correlations modelling the behaviours of walls “Wall1” and “Wall2” (Albidah, 2016; Altheeb, 2016) using the same constitutive and material models as for the numerical walls analysed in this paper. Hence the results from VecTor2 are considered reliable when making predictions of the actual yield and plastic displacements of a wall. Therefore, two RC walls analysed in VecTor2 are also used for comparison. The two FE walls are similar to that analysed in Section 3 but with a L_w of 3 m and a H_e of 7350 mm. The equations that have been proposed above to estimate the wall displacement are summarised in Appendix A. The set of equations used is dependent on the longitudinal reinforcement ratio and whether it is smaller or larger than the specified minimum value (Equation 4). Moreover, the equations used for comparison that are proposed by Priestley *et al.* (2007) are summarised in Appendix B.

Table 2 and Table 3 compare the predicted nominal yield and plastic displacement (respectively) from the proposed capacity equations to the observed experimental and analytical results. The resulting predictions of displacement capacity using the equations recommended by Priestley *et al.* (2007) are also given in Table 2 and 3, which also require a moment-curvature analysis to be undertaken for each of the wall sections. Table 4 gives the summation of the yield and plastic deformations, which corresponds to the ultimate displacement. It should be noted that the ultimate displacement is taken at the peak of the force-displacement response of the walls; this typically corresponds with an ultimate concrete strain of 0.003 being reached in the unconfined walls, a value which has been used by other researchers (Menegon *et al.*, 2015a; Tjhin *et al.*, 2004; Wood, 1989).

As shown in Table 2, the estimates of the nominal yield displacement made using the proposed equations are conservative for the experimental walls from Altheeb (2016), but are within 83% to 99% of the observed value for the walls analysed in VecTor2. The equations from Priestley *et al.* (2007) overestimate the nominal yield displacement, as was found in Section 3. It is estimated in Altheeb (2016) that 38% of the deformation at yield for Wall1 is due to flexural deformations, hence the use of Equations 6 and 7, which only calculate elastic deformation due to strain penetration, gives a conservative result. Similarly, it is estimated that flexural deformations are only responsible for 60% of the total displacement at yield for Wall2 (Altheeb, 2016), which corresponds closely to the value obtained in Table 2 using the equations proposed in this study (Appendix A).

Table 2 Nominal yield displacement results

Wall Specimen	ρ_{wv}	$\rho_{wv.min}$	Behaviour?	Nominal Yield Displacement (Δ_y)			Ratio (predicted/observed)	
				Observed (mm)	This Study (mm)	Priestley <i>et al.</i> (2007)	This Study (mm)	Priestley <i>et al.</i> (2007)
Wall1 (Altheeb, 2016)	0.36%	0.47%	Single Crack	10	4	19	0.45	1.95
Wall2 (Altheeb, 2016)	0.73%	0.46%	Secondary Cracking	11	6	19	0.61	1.76
3 metre wall (VecTor2)	0.15%	0.50%	Single Crack	4	3	35	0.83	8.63
3 metre wall (VecTor2)	1.00%	0.50%	Secondary Cracking	18	18	35	0.99	1.92

Table 3 Plastic displacement results

Wall Specimen	ρ_{wv}	$\rho_{wv.min}$	Behaviour?	Plastic Displacement (Δ_p)			Ratio (predicted/observed)	
				Observed (mm)	This Study (mm)	Priestley <i>et al.</i> (2007)	This Study (mm)	Priestley <i>et al.</i> (2007)
Wall1 (Altheeb, 2016)	0.36%	0.47%	Single Crack	29	26	25	0.90	0.88
Wall2 (Altheeb, 2016)	0.73%	0.46%	Secondary Cracking	32	14	15	0.43	0.47
3 metre wall (VecTor2)	0.15%	0.50%	Single Crack	20	17	60	0.87	2.98
3 metre wall (VecTor2)	1.00%	0.50%	Secondary Cracking	22	18	29	0.81	1.30

Table 4 Ultimate displacement results

Wall Specimen	ρ_{wv}	$\rho_{wv.min}$	Behaviour?	Plastic Displacement (Δ_p)			Ratio (predicted/observed)	
				Observed (mm)	This Study (mm)	Priestley <i>et al.</i> (2007)	This Study (mm)	Priestley <i>et al.</i> (2007)
Wall1 (Altheeb, 2016)	0.36%	0.47%	Single Crack	39	30	44	0.79	1.14
Wall2 (Altheeb, 2016)	0.73%	0.46%	Secondary Cracking	43	20	34	0.48	0.79
3 metre wall (VecTor2)	0.15%	0.50%	Single Crack	24	21	94	0.86	3.92
3 metre wall (VecTor2)	1.00%	0.50%	Secondary Cracking	40	36	63	0.89	1.58

For walls that are predicted to form secondary cracks, the large discrepancy of calculated plastic deformation compared to the observed experimental deformation for the wall with secondary cracking (Wall 2) could be due to the large amount of strain penetration deformations contributing to the overall global displacement; Altheeb (2016) reported that the deformation contributions at the peak displacement were from flexural (54%), strain penetration (36%) and shear (or sliding shear) (10%).

For the walls with single-crack failures that have rotations at the base driven by strain penetration rather than flexural deformations over a specified length, a good match is obtained for the plastic deformation using Equation 13 and a plastic hinge length of 150mm. In contrast, the equations by Priestley *et al.* (2007) widely overestimate the plastic deformation for the 3 metre FE wall with a single crack, primarily due to the calculated L_p of 1033 mm (Appendix B). Moreover, the nominal yield and plastic deformation predictions by Priestley *et al.* (2007) decrease in accuracy as the effective height (H_e) increases. This is due to the high dependency on H_e in the equations for calculating nominal yield and plastic displacement given in Equation 2 and Equation 14 respectively.

Overall, a closer match was achieved using the equations proposed here (in the Appendix A) when predicting the yield and plastic deformations of these lightly reinforced and unconfined rectangular walls that are common to the low-to-moderate seismic region of Australia.

7. Conclusion

This paper and corresponding study aimed to provide better estimates of the (nominal) yield and plastic deformation capacities for lightly RC walls. It was of particular importance to predict the displacement capacity of walls that exhibit a single crack, where the performance is predominantly governed by strain penetration deformation rather than the conventional flexural deformations. Nominal yield curvature and displacement coefficients were derived from a series of FE modelling analyses and have been included in the proposed displacement capacity equations for these types of walls; these are given in Appendix A. The equations have been shown to make conservative predictions when compared with available experimental results, and also with results from case studies that have been analysed using a comprehensive FEM analysis. The equations from Priestley *et al.* (2007) that have previously been used to make predictions for RC walls in regions of high seismicity have been shown to provide large overestimates of the displacement capacity in some case, particularly for large values of the H_e (e.g. mid and high-rise buildings) and walls that fail with a single crack. This work is part of ongoing research being conducted at the University of Melbourne to derive fragility functions for RC wall and core buildings in Australia.

8. Acknowledgment

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Appendix A

Summary of Displacement Capacity Equations (Assessment)

For $\rho_{wv} < \rho_{wv.min}$

$$\Delta_y = \Delta_{sp.e} = \frac{\Delta_{slip.elastic}}{0.5L_w - d_c} H_e = \frac{\varepsilon_{sy} f_{sy} d_{bl} H_e}{1.2(d_{bl})^{0.5} \sqrt{f_c'} (0.5L_w - d_c)}$$

$$\Delta_p = L_p(\phi_u) H_e = (150) \left(\frac{0.6\varepsilon_{su} - \varepsilon_{sy}}{L_w} \right) H_e$$

For $\rho_{wv} \geq \rho_{wv.min}$

$$\Delta_y = k_{\Delta} \phi_y H_e^2 = (39\rho_{wv} - 0.12) \left(\frac{1.6\varepsilon_{sy}}{L_w} \right) H_e^2$$

$$\Delta_p = L_p(\phi_u - \phi_y) H_e = ((0.10L_w + 0.075H_e)(1 - 6ALR) + L_{sp}) \left(\phi_u - \frac{1.6\varepsilon_{sy}}{L_w} \right) H_e$$

Appendix B – Equations proposed in Priestley *et al.* (2007)

$$\Delta_y = \frac{\varepsilon_{sy} H_e^2}{L_w} \left(1 - \frac{H_e}{3H_n} \right)$$

$$\Delta_p = L_p(\phi_u - \phi_y) H_e$$

$$L_p = 0.08H_e + 0.1L_w + 0.022f_{sy}d_{bl}$$

Appendix C – VecTor2 Results

Table C1 Results from VecTor2 for the lightly reinforced concrete walls

L_w (m)	ALR	H_e (m)	ρ_{wv}	VecTor2						
				Δ'_y (mm)	F_u/F_y	Φ_y (rad/mm)	$\Phi_y=1.6\varepsilon_{sy}/L_w$	$\Phi_y=2\varepsilon_{sy}/L_w$	Δ_y (mm)	k_{Δ}
3	1.5%	12.25	0.60%	17	1.31	1.5	1.4	1.8	22	0.10
3	1.5%	12.25	0.75%	33	1.21	1.4	1.4	1.8	40	0.18
3	1.5%	12.25	0.90%	40	1.22	1.5	1.4	1.8	49	0.22
3	10.0%	12.25	0.60%	24	1.29	1.6	1.4	1.8	31	0.14
3	10.0%	12.25	0.75%	39	1.14	1.6	1.4	1.8	45	0.20
3	10.0%	12.25	0.90%	44	1.15	1.5	1.4	1.8	51	0.23
6	1.5%	19.6	0.60%	17	1.44	0.8	0.7	0.9	24	0.09
6	1.5%	19.6	0.75%	40	1.22	0.6	0.7	0.9	49	0.17
6	1.5%	19.6	0.90%	50	1.23	0.6	0.7	0.9	62	0.22
6	10.0%	19.6	0.60%	22	1.51	0.6	0.7	0.9	33	0.12
6	10.0%	19.6	0.75%	48	1.17	0.7	0.7	0.9	56	0.20
6	10.0%	19.6	0.90%	55	1.16	0.8	0.7	0.9	64	0.23
9	1.5%	34.3	0.60%	28	1.61	0.3	0.5	0.6	45	0.08
9	1.5%	34.3	0.75%	87	1.22	0.4	0.5	0.6	107	0.19
9	1.5%	34.3	0.90%	100	1.25	0.4	0.5	0.6	125	0.22
9	10.0%	34.3	0.60%	44	1.52	0.4	0.5	0.6	67	0.12
9	10.0%	34.3	0.75%	98	1.17	0.4	0.5	0.6	115	0.20
9	10.0%	34.3	0.90%	111	1.18	0.5	0.5	0.6	131	0.23