

Issues for Seismic Hazard Analysis in Regions of Low to Moderate Seismic Activity

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ABSTRACT: There is a large degree of uncertainty regarding many aspects of seismic hazard analysis in regions of low to moderate seismic activity (stable continental regions, or SCR's) where earthquake occurrence is poorly understood and few strong ground motion recordings are available to constrain ground motion estimates. This uncertainty is represented by the existence of alternative models put forth by different experts. These epistemic uncertainties pertain to alternative distributed earthquake source models; alternative approaches to including active faults as earthquake sources; alternative models for the recurrence of earthquakes on these sources and active faults; and alternative ground motion prediction models. In probabilistic seismic hazard analysis, the epistemic uncertainty in each of these aspects should be treated by giving weight to all viable alternative models using logic trees, rather than by just using a preferred model. We illustrate these issues using Australia as an example of an SCR.

1 INTRODUCTION

There is a large degree of uncertainty regarding many aspects of seismic hazard analysis in Stable Continental Regions (SCR's), which we define as regions of low to moderate seismic activity remote from plate boundaries. This is because earthquake occurrence is often poorly understood in SCR's and there are usually few strong ground motion recordings available to constrain ground motion estimates. Earthquake forecasts in SCR's are usually based on distributed source models, because there are few if any identified active faults on which to base earthquake forecast models. However, in some SCR's including Australia, active faults contribute significantly to the hazard at low probability levels. The focus of this paper is on alternative procedures for developing earthquake forecasts and recurrence relations to represent both distributed earthquake sources and active faults, and on alternative ground motion prediction models for use in Australia. We summarise the implications of these issues for seismic hazard estimation, and propose practical approaches for addressing them in seismic hazard analysis in Australia.

2 DISTRIBUTED EARTHQUAKE SOURCES

2.1 Alternative Approaches to Modelling Distributed Earthquakes

The most common approach to representing distributed earthquakes is to use geological and geophysical criteria in combination with historical seismicity to identify zones of uniform seismic potential, and then use historical seismicity to characterize the seismic potential of each zone by means of the a-values and b-values of the Gutenberg-Richter earthquake recurrence model, together with an estimate of the maximum magnitude of earthquakes in each zone (Figure 1). This approach has the advantage of allowing for the incorporation of geological and geophysical information as well as seismicity data in the identification of seismic source zones. However, judgment is required in defining source zone boundaries, with the result that different investigators may develop quite different source models. Further, there is usually no physical basis for the resulting abrupt changes in seismicity levels across source zone boundaries, which may result in artificial features in hazard maps. This can be ameliorated by transitioning seismicity rates across the boundaries of the areal source zones (e.g.). Further, in areas of low seismicity the use of strain rate data can be useful for constraining seismicity rates in lieu of poor earthquake catalogues (e.g. Burbidge, 2009, Estimating the Rate of Deformation of Australia for the National Earthquake Hazard Map, AEES Conference)

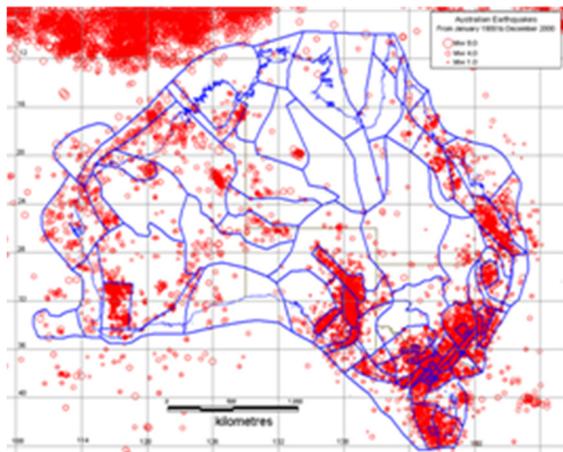


Figure 1. Historical seismicity and source zones. Source: Brown and Gibson, 2004.

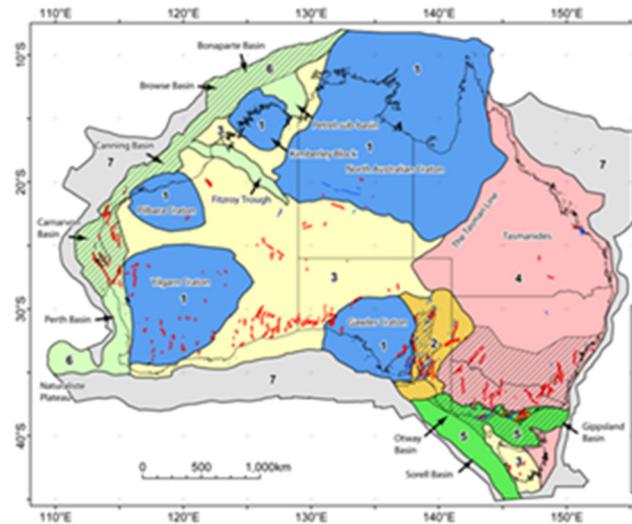


Figure 2. Neotectonic features and tectonic domains of Australia; cratons are shown in blue. Source: Clark et al. (2011).

These considerations motivate the use of spatially smoothed historical seismicity to generate the earthquake forecast. This approach was used to describe the seismic potential of the eastern United States in the U.S. National Probabilistic Seismic Hazard Maps (Frankel et al., 2007). This approach gives a spatially continuous source model without boundaries, with the possible exception of zones having different b-values (e.g. Hall et al., 2007). The spatial smoothing approach has the advantages of simplicity and of avoiding uncertainty in the geological definitions of zones, but has the disadvantage of not making use of potentially informative geological data.

The distribution of earthquake magnitudes in these earthquake source zones is usually assumed to follow the Gutenberg-Richter model (Figure 3). However, these recurrence models are often poorly constrained for large magnitudes, because the magnitude of the largest historical earthquake (which is commonly less than Mw 6) is usually much lower than the maximum magnitude that is attributed to the source zone (which is commonly as large as Mw 7.5).

In Australia, the source zone approach is used by Brown and Gibson (2004) and by Burbidge et al. (2012), and the spatial smoothing method is used by Hall et al. (2007). Generally, these different approaches do not give rise to systematic differences in hazard levels, but they can produce significantly different seismic hazard estimates at individual locations.

2.2 Localised Seismic Sources

In the source zone approach described above, it is assumed that the seismicity is uniformly spatially distributed throughout the source zone. However, Burbidge et al. (2012) identified “Hotspots” that are local earthquake sources characterized by spatially concentrated earthquake activity. Most of these Hotspots are thought to have ongoing seismic activity, although some appear to have commenced during the last 50 years. Spatially averaging the Hotspot seismicity with that of its host seismic source zone would underestimate the hazard near the Hotspot and overestimate it elsewhere.

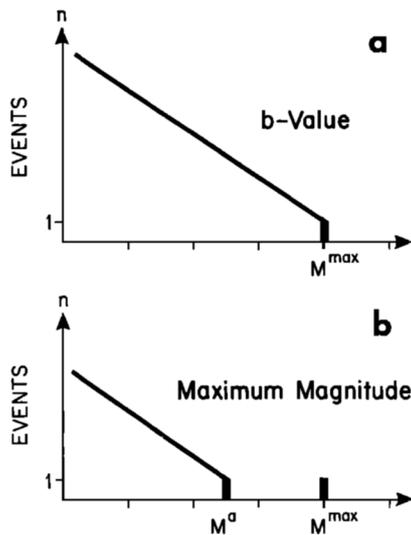


Figure 3. Gutenberg-Richter (top) and Characteristic interval recurrence models (bottom). Source: Wesnousky et al., 1983.

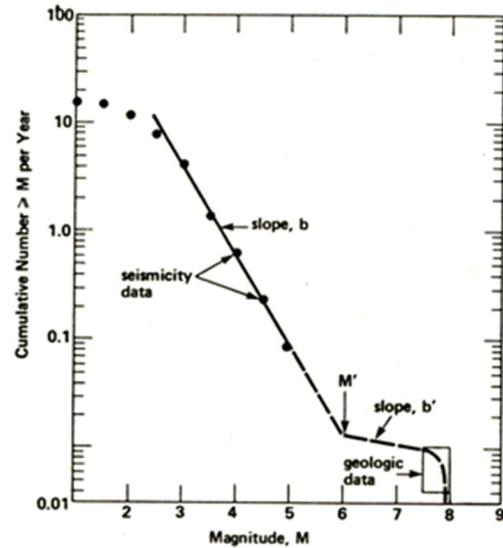


Figure 4. Characteristic cumulative earthquake recurrence model. Source: Schwartz and Coppersmith, 1984.

3 ACTIVE FAULT SOURCES

3.1 Introduction

In most regions, there are few if any identified active faults on which to base earthquake forecast models. Consequently, earthquake forecasts in these regions are usually based entirely on the distributed source models described above. However, where active faults are present, it is unclear how best to incorporate them in the earthquake forecast.

It is commonly thought that the recurrence relation for a sufficiently large region will include large active faults (whether they have been identified or not) and may therefore follow a Gutenberg-Richter recurrence model. According to this viewpoint, projecting the Gutenberg-Richter recurrence relation in a large region to magnitudes larger than the largest historical earthquake could provide a reliable estimate of the recurrence of larger earthquakes. This would also imply that including fault sources to the hazard would result in double counting because the historical seismicity, projected to larger magnitudes, already accounts for these larger earthquakes.

To avoid this potential double counting, Nis and Gibson (2006) proposed subtracting fault-related seismicity from the area source zone in which the fault occurs, and inserting a fault source having that seismicity, using a Gutenberg-Richter recurrence model. This approach assumes that the fault seismicity is represented in the background seismicity of the area source.

In another approach to avoid double counting, a 'pure characteristic model' could be used where a GR model is used for small to moderate magnitudes and then a characteristic model for larger magnitudes on faults which is larger than the background seismicity M_{max} (Stirling et al., 2012).

In contrast with the seismic zone approach, the approach used by Frankel et al. (1996) and Somerville et al. (2008) and described further below, is to assume that the distributed and fault sources of seismicity coexist independently. This approach combines distributed sources with fault sources whose seismicity is based on slip rate, without modifying the seismicity of the distributed source that hosts the fault, using a characteristic earthquake recurrence model. This approach assumes that the fault seismicity is not represented in the seismicity of the distributed source, in accordance with the characteristic earthquake recurrence model described below.

3.2 Characteristic Earthquake Recurrence Model

There is evidence that the distribution of earthquake magnitudes on discrete active faults may not be well represented by the Gutenberg-Richter model, and instead is better represented by the characteristic recurrence model, in which most of the fault slip is taken up in large earthquakes (Figure 4). The Characteristic recurrence model consist of the combination of a Gutenberg-Richter model for small magnitudes, derived from historical seismicity, and a characteristic model for large events derived from geological data. If the characteristic recurrence model applies, then the recurrence rate of large earthquakes may be underestimated by the Gutenberg-Richter model based on historical seismicity if it only contains small earthquakes. Moreover, if slip rate or recurrence interval estimates are available for the active faults, then their earthquake recurrence can be estimated directly from these slip rates or recurrence intervals, independently of the historical seismicity.

Estimation of recurrence models using geological data require decisions to be made as to the form and parameter values (e.g. maximum magnitude and transition magnitude of the Characteristic model) of recurrence relation that is used. It is commonly found that the frequency of occurrence of large earthquakes estimated from geological data is larger than that projected from the Gutenberg-Richter recurrence model for the seismic zone in which the active fault is located. However, this need not be regarded as an inconsistency, because the two estimates are based on very different data sets that each may have low predictive power in the other's domain. Some of the contrasts between these two data sets are described next.

3.3 Spatial Correlation of Historical Background Seismicity and Active Faults

It is commonly observed that low magnitude background seismicity is broadly distributed and not closely associated with active faults (e.g. in Figures 1 and 2). The presence of low magnitude background seismicity may therefore not be a sufficient condition for the occurrence of large earthquakes, which instead may be confined to discrete large faults. Conversely, the presence of low magnitude background seismicity may therefore not be a necessary condition for the occurrence of large earthquakes. For example, Clark et al. (2011, 2012) show the presence of surface faulting events in regions (such as southwestern South Australia, latitude 30°S, longitude 125 – 130°E) that have minimal background seismic activity (Figures 1 and 2).

3.4 Episodic Earthquake Recurrence on Faults

The apparent lack of correlation between low level background seismicity and active faults may be a manifestation of the very long recurrence intervals, on the order of tens of thousands of years, between large earthquakes on faults in Australia during their active phases, and of the apparently episodic nature of that activity, with intervals of seismic quiescence on the order of hundreds of thousands to millions of years separating active phases, which typically produce two or three large earthquakes, as suggested by Clark et al. (2011). In contrast, the historical record of earthquakes in Australia is very brief, a few hundred years at most, and is not nearly long enough to have adequately sampled the seismicity of Australia, especially at the larger magnitudes. To address this issue, Stirling et al. (2011) used a logic tree to partition the slip rate of faults into activity periods of inactivity and activity.

4 GROUND MOTION PREDICTION MODELS

4.1 Introduction

Brown and Gibson (2000) analysed the attenuation of recorded ground motion in eastern Australia from three earthquakes in the magnitude range of 4.8 to 5.1 in eastern NSW and Victoria and showed that it is more compatible with that for tectonically active regions than for tectonically stable regions. However, there are few ground motion recordings of earthquakes in Australia, and these are all from small magnitude earthquakes, so these data alone do not provide a direct means for developing ground motion models for Australia. Consequently, physics-based methods have been used to develop ground motion prediction models for Australia.

4.2 Ground Motion Models for Australian Earthquakes

Somerville et al. (2009) demonstrated their ability to simulate the recorded ground motions of small earthquakes that occurred in Eastern and Western Australia, and developed earthquake source scaling models for Australian earthquakes based on earthquake source modelling of the Mw 6.8 1968 Meckering the Mw 6.25, 6.4 and 6.6 1968 Tennant Creek earthquakes. They then used a broadband strong ground motion simulation procedure based on the elastodynamic representation theorem and Green's functions calculated from crustal structure models for various regions of Australia to calculate ground motions for earthquakes in the magnitude range of 5.0 to 7.5. These ground motions were then used to develop ground motion prediction equations, which were checked for consistency with available data from Australian earthquakes at each step. These ground motion models predict response spectra for two crustal domain categories: Cratonic Australia and Non-Cratonic Australia. The cratonic regions of Australia include much of Western Australia (but not the coastal strip west of the Darling Fault, including Perth); south-central South Australia; the northern part of the Northern Territory; and northwestern Queensland (Clark et al, 2011). Non-Cratonic Australia consists of the remainder of Australia, including Eastern Australia and part of the coastal margin of Western Australia, and includes all of the state capital cities.

Allen (2012) developed a ground motion model for southeastern Australia based on the stochastic finite-fault software package EXSIM (Motazedian and Atkinson, 2005; Atkinson and Boore, 2006). He first calibrated the parameters of the stochastic model using recordings of small earthquakes in southeastern Australia. The dependence of stress drop on earthquake depth was examined, and options were provided for variable stress parameter values in the ground motion prediction equation. The stochastic ground motion simulations were regressed to obtain model coefficients and the resulting ground motion prediction model was evaluated against recorded response spectral data for moderate-magnitude earthquakes recorded in southeastern Australia.

The Allen (2012) model provides a better fit to the distant recordings of small magnitude earthquakes in the Australian ground motion data base than does the Somerville et al. (2009) model, but the largest of those earthquakes has a magnitude of only 5.4. The Allen (2012) predicts unrealistically large ground motions close to large earthquakes, and is unconstrained by any such recorded ground motions. This shortcoming does not apply to the more physically-based simulations on which the Somerville et al. (2009) model is based, and which has been successfully used to simulate the near fault ground motions close to earthquakes with magnitudes up to 7.6 in California and Japan. This suggests that the magnitude scaling in the Somerville et al. (2009) model may be better calibrated against data than the Allen (2012) model at large magnitudes.

4.3 Comparison of Australian Ground Motion Models with Other Models

The left side of Figure 5 compares response spectrum predictions of the non-cratonic Australia model of Somerville et al. (2009) and the Southeastern Australia model of Allen (2012) for Vs30 of about 760 m/sec (NEHRP B/C boundary) with a corresponding prediction for tectonically active regions (Chiou and Youngs, 2014). It also shows the prediction for the same site condition in the SCR of eastern North America (Atkinson & Boore, 2006). These ground motions are in fairly close agreement for periods longer than about 0.1 sec.

The right side of Figure 5 compares response spectrum predictions of the cratonic Australia models of Somerville et al. (2009) and Liang et al. (2008) with corresponding predictions for the SCR for the Central and Eastern United States (CEUS) by Atkinson and Boore (1995) and Pezeschk et al. (2011) for hard rock site conditions (Vs30 of about 2000 m/sec). At short periods, all of these models except that of Liang et al. (2008) have much larger ground motions at short periods than those for soft rock conditions in southeastern Australia and tectonically active regions shown on the left side of Figure 5. The cratonic ground motion models for Australia of Liang et al. (2008) and Somerville et al. (2009) also have larger ground motions at periods longer than about 1 to 2 seconds. Using waveform modelling, Somerville and Ni (2010) showed that these large surface waves are due to the excitation of large Rg surface waves by shallow faulting in the craton, and are absent from the recordings of deeper earthquakes in southeastern Australia.

The Atkinson and Boore (2006) spectrum for soft rock conditions in the CEUS is shown on both the left and right sides of Figure 5 for reference. This helps demonstrate that the main difference between the ground motion models for soft rock sites in tectonically active regions (left side) and those for hard rock conditions in SCR's (right side) at short periods is attributable to differences in the shear wave velocity at the ground surface and to differences in source depth.

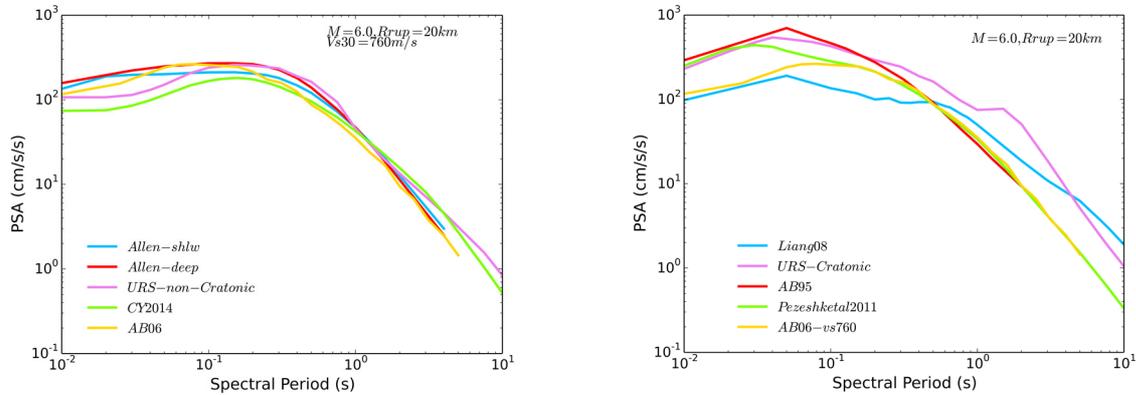


Figure 5. Response spectra for a magnitude 6 earthquake at a distance of 20 km. Left: Predictions for soft rock conditions: shallow and deep events of the Allen (2012) model for southeast Australia; the Somerville et al. (2009) model for non-Cratonic Australia, the Chiou and Youngs (2014) model for tectonically active regions, and the Atkinson & Boore model for the tectonically stable eastern North America. Right: Predictions for hard rock conditions: the Liang et al. (2008) and Somerville et al. (2009) model for Cratonic Australia, Atkinson and Boore (1995) and Pezeschk et al. (2011) for the CEUS, compared with the Atkinson and Boore (2006) model for soft rock conditions in the CEUS.

5 IMPLICATIONS FOR SEISMIC HAZARD ANALYSIS

5.1 Introduction

In this section we describe the implications of these issues for seismic hazard estimation, and propose practical approaches for addressing them in seismic hazard analysis.

5.2 Incorporation of both Seismic Source Zones and Active Fault Sources

As described above, in most SCR's there are few if any identified active faults on which to base earthquake forecast models. Consequently, earthquake forecasts in these regions are usually based distributed source models having Gutenberg-Richter recurrence relations. However, where active faults are present, it seems prudent to include them as separate additional earthquake sources, using Characteristic earthquake recurrence models, especially if estimates of their slip rates or earthquake recurrence are available.

Assuming that the largest earthquakes can only occur on these identified active faults, ideally we would like to limit the maximum magnitudes of the distributed earthquake source zones so that they do not duplicate the occurrence of large earthquakes on identified faults. However, from a practical viewpoint, it may be a long time before we are confident that all of the active faults have been identified, especially in view of their apparently episodic behaviour, and so it is simplest (although overconservative) to meanwhile use high maximum magnitudes for the distributed seismicity.

5.3 Non-Poisson Recurrence Models and Time-Dependent Seismic Hazard

As we have seen, the occurrence of large earthquakes in Australia is characterised by episodic behaviour that is not accurately represented by Poisson models, which have a random temporal distribution of seismicity. Also, as noted above, Burbidge et al. (2012) have identified Hotspots of localised seismic activity, most of which appear to be ongoing, but the activity of some of them has begun recently so it appears that at least some of them are non-Poisson features. The appropriate

treatment of these forms of non-Poisson behaviour in a probabilistic seismic hazard analysis depends on the annual probability level of interest. The results of probabilistic seismic hazard analyses are usually expressed as the hazard level that has a specified annual probability of exceedance. For example, in most countries, ordinary structures are designed for ground motions having an annual probability of exceedance of $1/475$, which is equivalent to a 10% probability in 50 years, and a return period of 475 years. The latter two expressions specifying time intervals of 50 and 475 years are sometimes misunderstood. The intent is to describe the instantaneous probability, and annual probability is used to do that. The expressions specifying time intervals of 50 and 475 years are only valid assuming Poisson models, and as we have seen, these may not apply for large earthquakes on active faults and for Hotspots. Modifications to conventional seismic hazard analysis methods are required to treat time-dependent hazard, and are not described further in this paper.

5.4 Increasing Hazard Contribution of Active Faults with Decreasing Annual Probability

Figure 6 shows hazard curves for peak acceleration (left) and 1 second response spectral acceleration at a site located close to several active faults with slip rates of up to 0.15mm/yr . Ordinary structures are usually designed for ground motions having an annual probability of exceedance of $1/475$, although there is a growing tendency for some level of performance to be checked at an annual probability of exceedance of $1/2475$. At annual probabilities of $1/475$ and $1/2475$, the active faults do not contribute significantly to the seismic hazard at the site. However, for an annual probability of $1/10,000$, the active faults make a significant contribution to the seismic hazard, reflecting the average recurrence interval of about 10,000 years for large earthquakes on them. At annual probabilities lower than about $1/25,000$, the faults dominate the seismic hazard at the site.

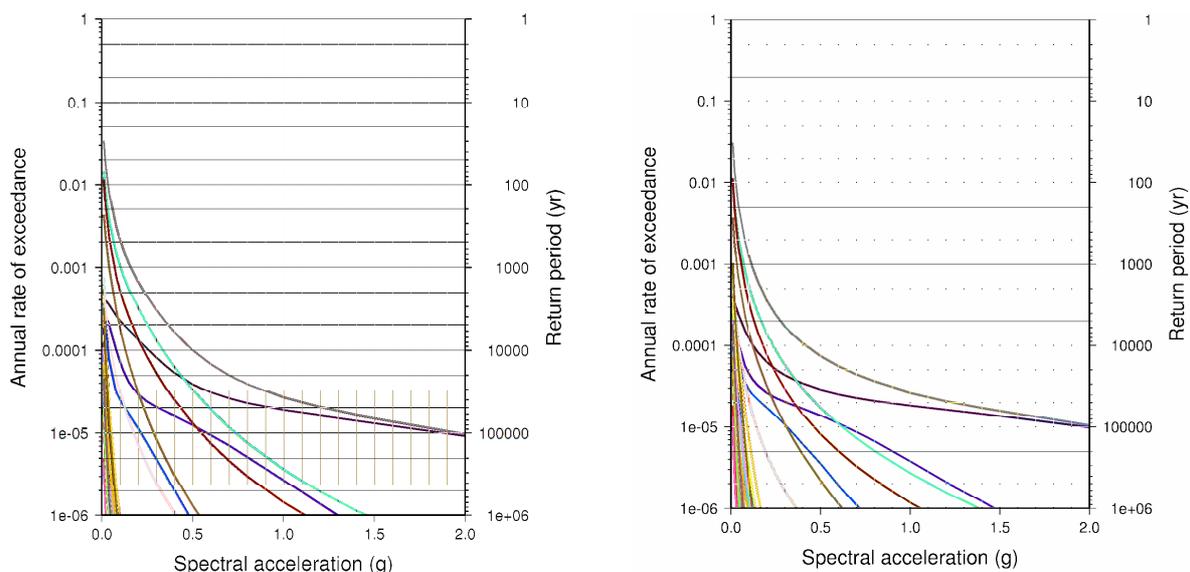


Figure 6. Hazard curves for peak acceleration (left) and 1 sec response spectral acceleration (right) for a site near a fault with a slip rate of 0.15 mm/yr . The black line is the curve for the closest fault, the two kinked blue lines are for other nearby faults, the green and brown lines represent distributed seismicity, and the grey curve represents the total hazard. Modified from Somerville et al., 2008.

5.5 Episodic Behaviour of Active Faults

We have seen that Clark et al. (2011) found that earthquake activity on faults in Australia is episodic, with clusters of earthquakes on a given fault occurring close together in time (several tens of thousands of years), separated by longer periods (several hundreds of thousands of years) of no large earthquake activity. Using the results of Clark et al. (2011), it may be possible to identify which faults are currently in an active phase and which are currently in an inactive phase. This could then be applied to the evaluation of the seismic potential of active faults in seismic hazard evaluations.

5.6 Ground Motion Prediction Models

The ground motion prediction models for Australia that have been developed in recent years by Allen (2012) and Somerville et al. (2009) are in reasonably close agreement with those for other tectonic regions, and also embody features that are specific to Australia. The use of Australian models in combination with other models provides a means for representing the large degree of epistemic uncertainty in ground motion prediction models for Australia. The NGA-East project has a large database for SCR ground motion (>11,000 recordings) and NGA models for Eastern North America, both of which could be used as a comparison with Australian models and data. (http://www.daveboore.com/pubs_online/webPEER-2015-04-NGA-East.chapters_1_2.pdf).

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