Estimation of Yield Curvature for Direct Displacement-based Seismic Design of RC Columns

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Abstract

Significant research efforts have been devoted in recent years to the development of displacement-based seismic design methodologies, recognizing the shortcomings of traditional, code-specified force-based design procedures. Recent advances in direct displacement-based seismic design of columns rely on the estimates of yield curvature for the determination of seismic design forces to satisfy the specified seismic performance levels. This paper presents simple expressions for estimating the effective yield curvature for normal- and high-strength circular reinforced concrete (RC) columns based on moment-curvature analyses of a large number of column sections. Such expressions can be programmed into the spreadsheet format and can be used for the displacement-based design of RC columns. Influence of different parameters on the effective yield curvature has also been quantified. Effective yield curvature is presented in terms of the gross diameter of the section and the yield strain of the longitudinal reinforcement together with three modification factors that take into account the effects of the compressive strength of concrete, the axial load ratio and the quantity of longitudinal reinforcement.

Keywords: seismic, design, yield, curvature, displacement, performance

1. INTRODUCTION

Earthquake engineering research over the past three decades resulted in force-based (FB) structural design and detailing procedures, incorporating capacity principles and ensuring adequate safety of the structure without significant damage and casualties when subject to severe ground shaking [1-3]. Certain fundamental assumptions in the FB design procedures have, however, been found to be not representative of actual structural behaviour. The stiffness of the structure, which is used for determining the natural period of the structure and the distribution of forces to different structural elements, is not known initially. Different codes take different approaches in estimating their stiffnesses. Moreover, the effective stiffness of a cracked concrete member is not constant but increases with increasing flexural strength [4]. Difficulties in determining the structural stiffness of concrete elements result in significant inaccuracies in the estimates of the natural period of the structure. These errors are then translated into errors in the estimation of the strength demand and the distribution of forces calculated from the FB design procedures.

Considering the inherent limitations of the FB procedures, displacement-based (DB) procedures have been proposed in recent years where displacement demand is compared with the displacement capacity of the structure. Such procedures are transparent as they consider the true behaviour of the structure and will likely be incorporated in future seismic design codes. In the DB procedures, the design force levels are based on the estimate of yield displacement of the structure. The ultimate displacement demand is first compared with the yield displacement for estimating the ductility demand of a cross-section. The section ductility demand can be used for calculating the effective damping level which is then used for calculating the inelastic displacement demand and the effective natural period based on the elastic displacement response spectrum [5]. The comparison between the yield displacement and ultimate displacement is also important for evaluating the seismic performance of a structure or structural element under a given level of earthquake shaking.

The intended mechanism for the majority of bridge piers and columns involves the formation of plastic hinges at critical locations (ie. plastic hinge regions). The yield displacement of concrete columns can be estimated using simple expressions that account for the flexural deformation of the column [4]. Such expressions can be developed based on the yield curvature of the column at the critical location. Previous studies on yield curvature [4, 6, 7] indicate that yield curvature is not sensitive to the quantity of longitudinal reinforcement used in the member. This allows simple expressions to be used for estimating the curvature of the member at yield. Once the yield curvature and the yield strength are known, the cracked stiffness of the member can be obtained readily.

This paper aims at developing simple expressions for estimating the yield curvature of normal- and high-strength circular RC columns based on moment curvature analysis of a large number of column sections.

2. PREVIOUS STUDIES ON YIELD CURVATURE

Priestley et al. [6], and later Priestley et al. [4], proposed a formula for calculating the yield displacement for circular bridge columns taking into account shear contribution and strain penetration of the longitudinal reinforcement into the foundation. Effective yield curvature (ϕ_y) has been expressed in terms of the yield strain of the longitudinal reinforcement (ε_{ys}) and the diameter of the gross section depth (D) of the piers (Equation 1). As mentioned earlier, the effective yield curvature of a cross-section does not depend significantly on the ratio of the longitudinal reinforcement; whereas, effective stiffness is almost proportional to the ratio of the longitudinal reinforcement (given that stiffness is strength divided by displacement at yield). Hence, the effective curvature should be considered as one of the basic properties of a cross-section. However, in the proposed formula no indication has been given on the sensitivity of the yield curvature to the ratio of the longitudinal reinforcement. Moreover, axial load ratio which may affect the yield curvature has not been parameterized and no limitation has been introduced for the application of the formula. However, bridge columns designed according to most design codes may have the axial load ratio (P/f'_cA_g; where P=axial load, f'_c=concrete compressive strength and A_g=gross area of the column) of around 10%.

$$\phi_y = 2.25 \times \frac{\varepsilon_{ys}}{D} \tag{1}$$

Montes and Aschleim [7] proposed simple expressions for the calculation of the effective yield curvature based on moment-curvature analyses. Yield curvature has been expressed in terms of the yield strain of longitudinal reinforcement (ε_{ys}) and the effective depth of the section, d (depth of the extreme tension reinforcement) as shown by Equations 2a,b.Less scatter is seen with the estimate of the yield curvature when the effective depth of the cross-section has been parameterised..

$$\phi_y = 2.4 \times \frac{\varepsilon_{ys}}{d} \quad \text{for steel } f_y\text{= 400 MPa} \tag{2a}$$

$$\phi_y = 2.3 \times \frac{\varepsilon_{ys}}{d} \quad \text{for steel } f_y\text{= 500 MPa} \tag{2b}$$

One of the most important observations from this study is the sensitivity of the yield curvature to the level of axial load (n) (Equations 3a, b).

$$\phi_y = \frac{\varepsilon_{ys}}{d}\left[2.5 - \left(a - b.\frac{P}{f_c' A_g}\right)^2\right] \quad \text{for steel } f_y\text{=400 MPa} \tag{3a}$$

$$\phi_y = \frac{\varepsilon_{ys}}{d}\left[2.4 - \left(a - b.\frac{P}{f_c' A_g}\right)^2\right] \quad \text{for steel } f_y\text{=500 MPa} \tag{3b}$$

Values of a and b have been found from parabolic curve fitting and have been expressed as functions of the effective depth for three cross-sections. For other cross-sections, it is recommended to interpolate the values of a and b.

As observed from the study by Priestley et al. [6] and Priestley et al. [4], there has been no indication on the sensitivity of the yield curvature to the quantity of longitudinal reinforcement. High-strength concrete with compressive strength of up to 100 MPa is now being increasingly used in the construction of bridge columns and is also permitted by most design codes. It appears that high strength concrete was not within the scope of the studies cited above (which have not explicitly parameterised concrete strength in its recommended expressions). The concrete cover has also not been parameterised.

3. MODELLING OF COLUMNS FOR YIELD CURVATURE

Analytical Modelling of RC Columns

Reinforced concrete is a highly non-linear material. Realistic constitutive law of reinforced concrete is complex as the non-linearity arising from concrete and the reinforcement needs to be appropriately combined to accurately simulate the experimentally observed behaviour of reinforced concrete elements.

Stress-Strain Relationship of Concrete

The uniaxial confined concrete model of Légeron and Paultre [8], which is based on strain compatibility and transverse force equilibrium, has been chosen as the constitutive law of concrete for the analytical modelling of RC columns. The model has been validated with test results from more than 200 circular and square large-scale columns tested under slow and fast concentric loading. In the model, the behaviour of confined concrete is related to the effective confinement index, which takes into account the amount of transverse confinement reinforcement, the spatial distribution of the transverse and longitudinal reinforcement, the concrete strength, and the transverse reinforcement yield strength.

Stress-Strain Relationship of Longitudinal Bars

An accurate model of a stress-strain relationship of steel bars needs to simulate: (i) elastic, yielding and strain hardening behaviour, (ii) compression behaviour including buckling of bars, and (iii) low cycle fatigue and premature rupture of bars in tension.

Gomes and Appleton [9] model has been chosen as the constitutive law of reinforcing bars, since it is simple and well predicts the above characteristics of reinforcing bars. The model takes into account the effect of inelastic buckling of longitudinal reinforcing bars in a simplified way based on the plastic mechanism of a buckled bar.

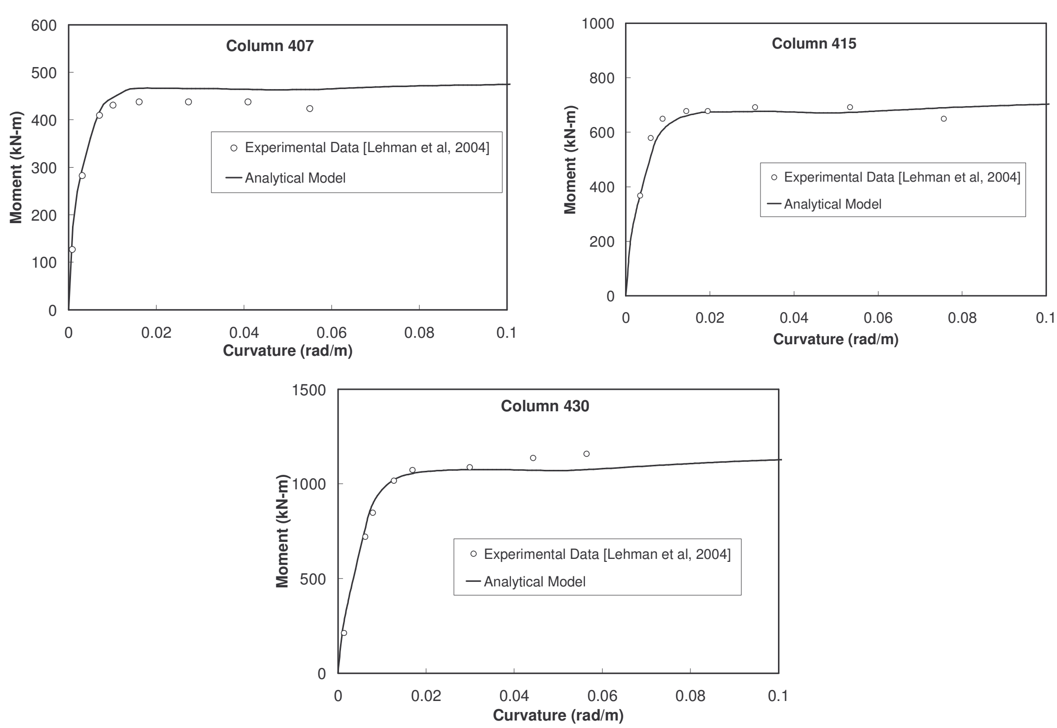

Figure 1: Experimental results compared with analytical predictions

Modelling Sectional Behaviour

In this study, the complete moment curvature responses of column sections were computed using computer program MNPHI [10] which has incorporated the constitutive laws of concrete and that of the reinforcing bars. The program uses a layered representation of the section where each layer is separated into a confined core layer and an unconfined cover layer with the corresponding material properties. It calculates the moment-curvature response by an incremental analysis assuming plane section remaining plane (before and after bending). The program also takes into account the spalling of the concrete cover.

Comparison with Experimental Results

To evaluate the capability of the developed analytical model, experimental results of a large number of columns tested under cyclic loading have been compared. Due to space restrictions, moment curvature predictions of bridge columns 407, 415 and 430 (reported by Lehman et al., [11]) are presented herein in Figure 1. Excellent agreement has been observed between the experimental results and analytical predictions from this study. The developed analytical model has been used for the development of moment curvature relationships for this study.

Definition of Yield Curvature

Different definitions of yield curvature have been found in the literature based on both experimental and analytical results. For the purpose of design of reinforced concrete columns, effective yield curvature rather than true yield curvature is of interest. Priestley et al. [6] defined the effective yield curvature as the intersection of the line through the first yield point with the line drawn tangent to the M-ϕ diagram. This definition of yield curvature can be useful in the presentation of the experimental results (when sufficient data are not available for reliably estimating the flexural strength of the concrete). However, there is an element of user subjectivity in fitting the tangent line as different tangent lines can be fitted to the softened branch of moment curvature curve. The definition adopted in this paper is based on the first yield point (either concrete or steel) and the maximum flexural strength of the column (Equation 4):

$$\phi_y = Min\left(\phi_{yc}\frac{M_{\max}}{M_{yc}};\phi_{ys}\frac{M_{\max}}{M_{ys}}\right) \tag{4}$$

where ϕ_{yc} is the curvature when the concrete strain reaches the peak stress of the unconfined concrete, ε_c', and ϕ_{ys} is the curvature at the onset of yielding of the longitudinal reinforcement. The definition of yield curvature adopted herein (Figure 2) avoids user subjectivity as it is based on the maximum flexural strength of the column. This allows engineers to develop bi-linear moment curvature response functions which are of practical importance to the structural design of reinforced concrete columns.

4. ESTIMATION OF YIELD CURVATURE

Expressions for the effective yield curvature presented in this paper are based on the moment-curvature response of columns according to methodology developed in Section

3. It was observed that the effective yield curvature of the column is influenced by the overall size of the cross-section (diameter) D, the axial load ratio, n, the strength of concrete, f'_c and reinforcement, f_y, and to some extent on the ratio of longitudinal reinforcement, ρ and the depth of cover concrete, c (or equivalently the ratio of area of the gross section to the area of the core, A_g/A_c).

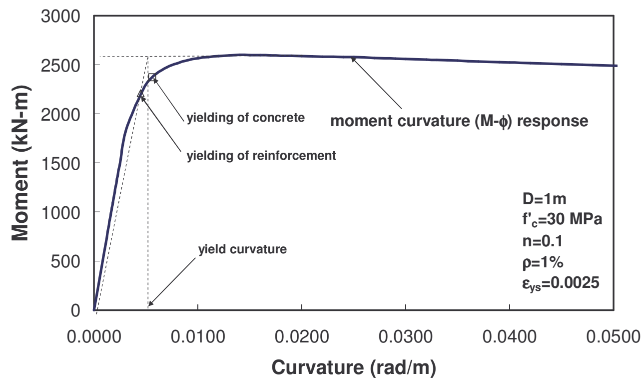

Figure 2: Moment curvature (M-ϕ) response

Circular column sections of diameter from 0.5-2.5 m having axial load ratio of 0-0.5 and reinforcement ratios of 1-6% have been considered. Both normal- and high-strength columns were within the scope of the study. Concrete strengths ranging from 30 MPa to 100 MPa, which covers the wide range of concrete currently used for the design of concrete columns and also the maximum limit permitted by most design codes. Concrete cover has been considered as 0.05 m. However, to study the influence of the concrete cover on the effective yield curvature of the columns, cross-sections with concrete cover varying between 0.03m and 0.1m were analysed. The yield strength of reinforcing bars was assumed to be 400 MPa, 500 MPa and 600 MPa. The elastic modulus of the reinforcing bar was taken as 200 GPa.

Influence of Section Dimension

Previous researches on yield curvature proposed expressions for yield curvature in terms of overall cross-section dimension [4,6] and also in terms of the effective depth of the cross-section [7]. In this paper, the effective yield curvature is presented in terms of the overall cross-section dimension (D).

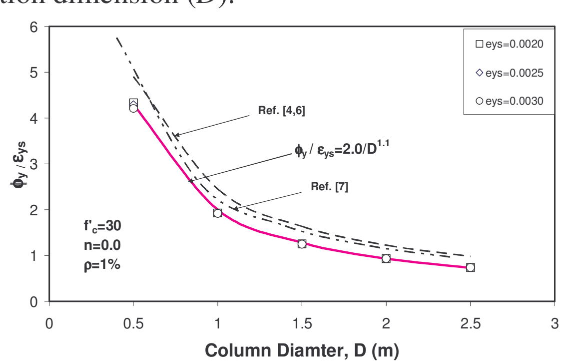

Figure 3: Influence of column diameter

It is evident from Figure 3 that the effective yield curvature is significantly dependent on the diameter of the cross-section (D) and that the best fit curve can be obtained when it is expressed in term of $D^{-1.1}$. The following expression is proposed for the estimation of the effective yield curvature to design normal strength (f'_c= 30 MPa) concrete columns whilst ignoring the effects of the axial load (n=0).

$$\phi_y = 2.0 \times \frac{\varepsilon_{ys}}{D^{1.1}} \tag{5}$$

It is seen from Figure 3 that previous studies have over-estimated the effective yield curvature, especially when the diameter of the section is small. This over-estimation is also apparent from results presented in Ref. [7].

Intuitively, the ratio of area of the gross section to the area of the core (A_g/A_c or the thickness of concrete cover) may have some influence on the estimates of the effective yield curvature. Figure 4 presents the influence of concrete cover on the estimation of effective yield curvature. All data points have been normalized with respect to the yield curvature when the concrete cover is 0.05 m.

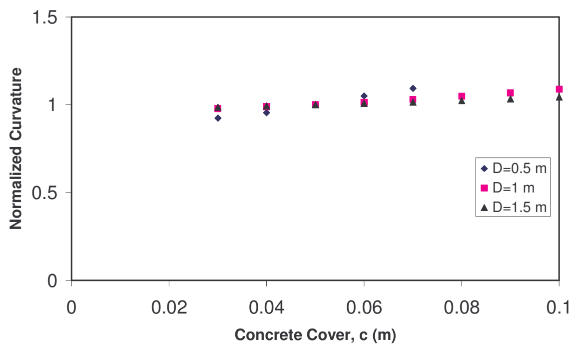

Figure 4: Influence of the concrete cover

It is evident from Figure 4 that concrete cover (or A_g/A_c) does not have significant influence on the effective yield curvature. The influence is even less when the overall diameter of the cross-section is greater than 1m. However, the concrete cover may have some effects, though not significant, when the diameter of the section is less than 0.5 m. In Figure 4, concrete cover of up to 0.07 m has been considered for a column of 0.5 m in diameter. It is apparent that the gross diameter of the cross-section is a better parameter to use than the effective depth of the section.

Influence of the Strength of Concrete

Figure 5 presents the influence of the concrete strength (f'_c) on the effective yield curvature. The normalized yield curvature (normalized with respect to the yield curvature of 30 MPa concrete) varies from 1.0 to 0.9 when the concrete strength varies from 30 MPa to 100 MPa. Thus, ignoring the effects of concrete strength could only result in the overestimation of the effective yield curvature by up to 10% only (when the concrete strength is 100 MPa). However, the strength of concrete has significant influence on the yield curvature and the axial load ratio (refer next sub-section). A modification factor has been proposed to take into account the effects of the axial load level (Equation 6).

$$MF(f'_c) = 1.25 \times f_c'^{-0.07} \quad (6)$$

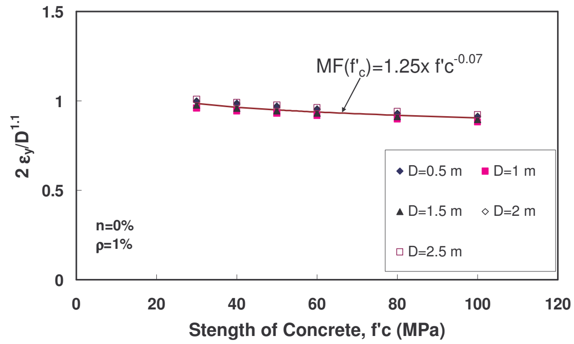

Figure 5: Influence of Strength of Concrete

Influence of Axial Load

Figure 6 presents the influence of the axial load ratio on the normalized yield curvature (normalized with respect to the yield curvature at zero axial load level). The data points shown in the figure represent the average values of the normalized yield curvature for all the section dimensions considered (0.5m-2.5m). It is seen that concrete strength influences the yield curvature when subject to varying axial load ratio. Yield curvature is shown to increase with increasing axial load ratio from 0-0.3 and beyond that point effective yield curvature decreases with increasing axial load ratio. Similar observations can be found in Ref. [7]. A modification factor has been proposed to take into account these observations (Equation 7).

$$MF(n) = 1 + \left(0.041 \times f'_c - 0.26\right) \times n - (0.043 \times f'_c + 0.85) \times n^2 \quad (7)$$

where n is the axial load ratio and f'_c is the strength of concrete. It is important to mention that such modification factor may overestimate the yield curvature for smaller diameter section ($D < 0.5$ m) under high axial load. However, bridge piers are normally subjected to low level of axial load (around 10%) and usually have larger diameter cross-sections.

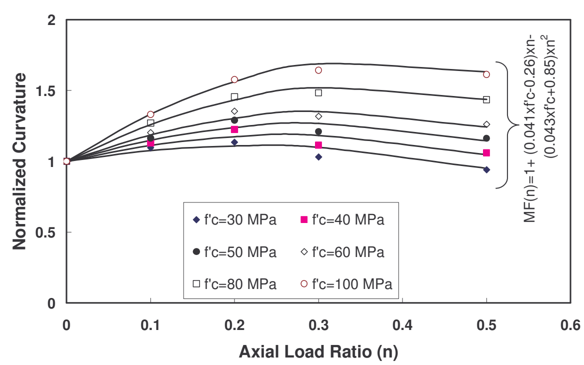

Figure 6: Influence of Axial Load

Influence of Longitudinal Reinforcement Ratio

Previous studies on yield curvature have not explicitly studied the effect of axial load ratio as mentioned in Section 2. It has been argued that longitudinal reinforcement ratio does not have significant influence on the effective curvature. Figure 7 presents the influence of longitudinal reinforcement ratio on the normalized effective yield curvature (normalized with respect to the yield curvature when longitudinal reinforcement ratio=1%). The effect of longitudinal reinforcement is very low when the longitudinal reinforcement ratio is more than 3%. In most bridge design codes, the permitted amount of longitudinal reinforcement varies from 1-6%, although in real practice longitudinal reinforcement ratio less than 3% is usually provided to avoid congestion of the reinforcement.

Results shown in Figure 7 are based on the condition of no axial load and the difference would be even lower if the axial load level is considered (i.e. $n > 0$). A modification factor has been proposed to take into account effects of the longitudinal reinforcement ratio (Equation 8).

$$MF(\rho) = \rho^{0.16} \qquad (8)$$

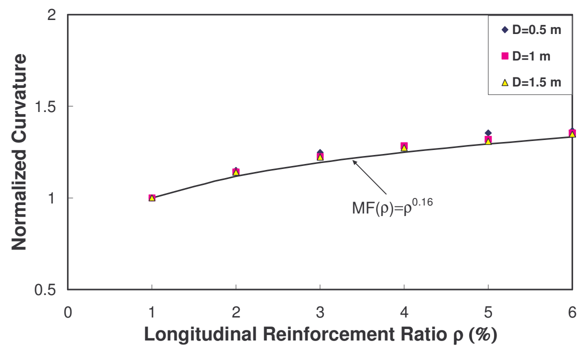

Figure 7: Influence of the Longitudinal Reinforcement Ratio

Proposal for the Estimation of Effective Yield Curvature

Based on the parametric studies conducted on more than 200 columns, algebraic expressions have been developed (Equation 9) for the estimation of the effective yield curvature. It was found that ignoring the effects of the longitudinal reinforcement ratio would only marginally underestimate the effective yield curvature and hence its effects could be neglected in the preliminary design of the concrete column. However, parameterising the effects of longitudinal reinforcement ratio can be useful for the accurate evaluation of existing concrete columns.

$$\phi_y = 2.0 \times \frac{\varepsilon_{ys}}{D^{1.1}} \times MF(f_c') \times MF(n) \times MF(\rho) \qquad (9a)$$

$$MF(f_c') = 1.25 \times f_c'^{-0.07} \qquad (9b)$$

$$MF(n) = 1 + \left(0.041 \times f_c' - 0.26\right) \times n - (0.043 \times f_c' + 0.85) \times n^2 \qquad (9c)$$

$$MF(\rho) = \rho^{0.16} \qquad (9d)$$

The above equations can be easily programmed into an excel spreadsheet for estimating the effective yield curvature for a wide range of concrete columns having different axial load ratios, concrete strengths and longitudinal reinforcement ratios.

Figure 8 indicates that the predicted values of the yield curvatures are within 10% of the calculated yield curvatures. Several data points have been observed to be significantly different from the calculated values. These data points are for 0.5 m diameter columns with axial load ratios of between 0.3 and 0.5. It is mentioned earlier that the $MF(n)$ may overestimate the yield curvature for small diameter columns under higher axial load ratio. However, the conditions of high axial load ratios in small diameter columns are not common in bridge construction.

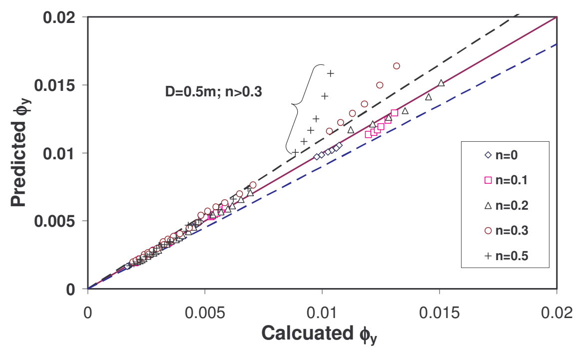

Figure 8: Calculated Versus Predicted Yield Curvature

5. CONCLUSIONS

Simple expressions for the estimation of the effective yield curvature for normal- and high-strength circular reinforced concrete (RC) columns have been developed based on moment curvature analyses of 200 column sections. Such expressions can be easily programmed into an excel spreadsheet and can be very useful for the preliminary design of concrete columns and also for the performance evaluation of existing columns.

Previous studies on the effective yield curvature have been reviewed critically and the limitations of the studies in incorporating the influence of axial load, strength of concrete and reinforcement have been identified. The present study is a significant improvement over the previous studies as all the parameters that influence the estimate of yield curvature have been quantified.

The yield curvature is influenced by the size (diameter) of the section, the axial load ratio, the strength of concrete and to some extent the amount of longitudinal reinforcement and the thickness of the concrete cover. It has been observed that the best estimate of yield curvature can be obtained when it is expressed in terms of the gross section depth rather than the effective depth, as the concrete cover has insignificant influence.

The amount of longitudinal reinforcement does not have significant influence on the effective yield curvature. Hence, the effective yield curvature can be estimated reasonably without explicitly considering the influence of the amount of longitudinal reinforcement. This is especially useful for the preliminary design of concrete columns. However, the effect of longitudinal reinforcement may be important for the accurate performance evaluation of existing columns.

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