

# MODELLING OF EARTHQUAKE INDUCED OVERTURNING OF BUILDING CONTENTS

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## ABSTRACT

Damage to critical contents in buildings following an earthquake (or explosion) often results in significant economic losses if the building cannot perform its normal functions over an extended period of time. Building codes have begun to address potential damage to building contents, but this has been handicapped by the lack of knowledge on their potential behaviour in an extreme event. The conventional force-based approach of estimating the peak floor acceleration and the associated inertia forces on an object is unrealistic, as the components are often unrestrained. Clearly, the normal assumption that "*building content is part of the building's primary structure (i.e. fixed components)*" is inappropriate.

This paper introduces a rational procedure to model the rocking response behaviour of free-standing objects when subjected to earthquake excitations. This new method utilizes the displacement-based approach and presents results in the form of Rocking Displacement Spectra (RDS). The RDS is different to the Displacement Response Spectra in that, the displacement demand is calculated from non-linear time-history analyses and results are expressed as a function of the object dimensions (i.e. thickness and height) as opposed to the object natural period. Given a certain excitation on a building floor, the RDS provides a direct indication of the maximum displacement experienced by a range of components, and quantifies their risk of overturning. The RDS format represents a significant step forward in modelling overturning of building contents.

## 1. INTRODUCTION

The development of analytical techniques in current buildings standards (including the Australian Earthquake Standard, AS 1170.4, 1993), for free standing objects has mirrored that for the primary structure of buildings. Most of these techniques use equivalent lateral force methods (i.e. force based approach), where the components are designed for a lateral seismic force that is expressed as the product of the component mass and the Peak Floor Acceleration (PFA) developed in the component according to basic principles of mechanics. The PFA at any level in the building is obtained by linear interpolation between acceleration estimated at the roof and ground level (e.g. FEMA 356, 2000; IBC, 2000 and AS 1170.4, 1993). The value of the design PFA at the roof of the building may be estimated by taking the ratio of the estimated inertia force acting at the roof level and the mass of the roof.

The objective of this conventional force based design approach is to produce an anchorage or bracing scheme for the components that can withstand the acceleration generated by the earthquake, without allowing the component to shift or topple. Accordingly the analytical techniques for assigning the response of components, which are demonstrated by recent building standards, assume that components would be fully restrained.

In regions of low to moderate seismicity including Australia, the provision of holding-down fasteners to secure building contents is not common. Computer cabinets and electrical components are typically free-standing and without any holding-down fasteners (Al Abadi *et al*, 2003). Enforcing full restraints on every item of equipment in every facility is desirable but has never been implemented in Australia, and yet failure or damage to building contents in an earthquake (or explosion) could disturb the continuous functioning and subject its occupants to significant risks even in a moderate earthquake. Hence, this paper focuses on the response of free-standing objects, which are free-standing and without effective hold-down connections.

The concept of using displacement spectra (i.e. displacement based DB approach) to model the behaviour of the unrestrained component is introduced in this paper by a new rational approach, which presents results in the form of Rocking Displacement Spectra (RDS).

The RDS is a plot of the maximum object displacement, versus the parameter  $R$  (distance between the object centre of mass to the pivotal edge) for different object thicknesses.

Given the basic parameters of an object, RDS can be used directly to predict whether components will overturn or rock. And for the rocking scenario, RDS can predict the component maximum displacement.

In this paper, a brief review for the Elastic Displacement Spectra approach is presented in Section 2. Section 3 presents the background and development of the Rocking Displacement Spectra (RDS) approach.

## 2. ELASTIC DISPLACEMENT SPECTRA (EDS) APPROACH

It has been found from a recent investigation (Al Abadi *et al*, 2004) into the response of slender free-standing rectangular objects that the maximum displacement developed in a rocking motion can be reasonably approximated by the motion predicted for a Single-Degree-Of-Freedom (SDOF) lumped mass system, as shown schematically in Fig. 1a.

The force-displacement relationship representing the response to a push-over load is shown in Fig. 1b (refer line drawn in bold). Using substitute-structure modelling, the force-displacement behaviour could be linearized with an effective stiffness. Accordingly the effective period (or  $T_{eff}$ ) of rocking could be estimated by the formula shown in Fig. 1. It should be noted that the highest point on the displacement spectrum does not necessarily appear as a prominent peak when plotted in the conventional acceleration response spectrum format. Thus, the described approach of tracking the maximum displacement demand from a displacement spectrum would not have been possible using the acceleration response.

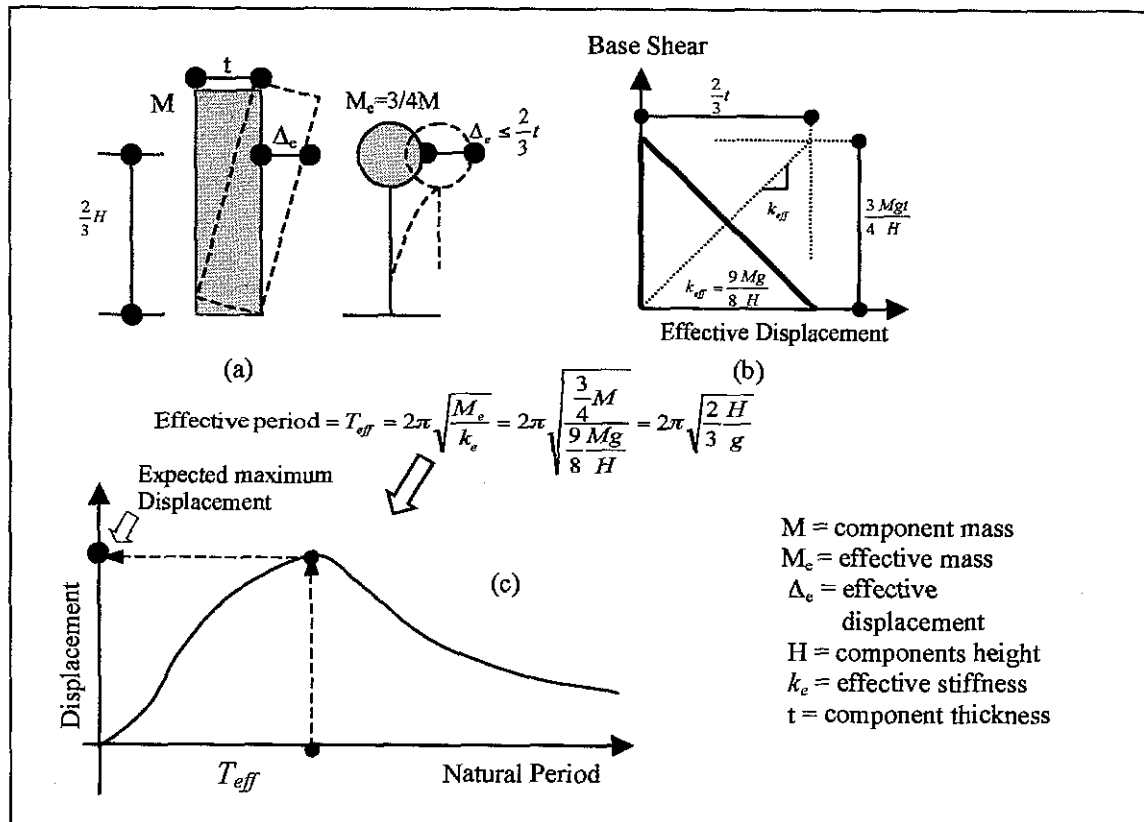


Fig. 1: Elastic displacement spectra for rocking response.

This simplified method of using an elastic displacement spectrum to predict the displacement of a rocking object by substitute-structure modelling, is only meant to provide an approximation to the actual response. The modelling is particularly susceptible to significant errors when the response is highly non-linear. This is discussed further in the following section.

### 3. ROCKING DISPLACEMENT SPECTRA (RDS) APPROACH

Rocking response for rigid bodies had been subjected to extensive study, as reviewed by Makris and Roussos (1998). This paper presents only the development of the rocking spectra concept. Research on the development of rocking spectra has recently been conducted by Makris and Konstantinidis (2001) in studying the response of ground mounted electrical transformers. In their study, the proposed rocking spectra were plots

of the maximum rotation,  $\theta$ , versus the frequency parameter,  $p$  (or its inverse "Time period,  $T=2\pi/p$ ").

The Rocking Displacement Spectra (RDS) approach is presented herein as a function of the object dimensions (i.e. thickness, height) as opposed to the object natural period. To show the development of RDS, a rigid block as shown in Fig. 2 is considered. Two main assumptions are applied to the model, which are: (i) the magnitude of horizontal acceleration is sufficiently large enough to cause initial rocking; (ii) the coefficient of friction is large enough so that there is no sliding. Complex motions such as "walking" have been ignored at this stage of the investigation. Under a positive horizontal acceleration that is sufficiently large, a rigid block will initially rotate with a negative rotation,  $\theta < 0$  and, if it does not overturn, it will eventually assume a positive rotation; and so on. Uniform distribution of mass is also assumed so that the centre of mass of the object is also at its geometric centre. Investigation is continued to address objects with non-uniform distribution of mass.

The equations that govern the rocking motion under horizontal ground/floor acceleration  $\ddot{u}_g(t)$  are

$$I_o \ddot{\theta}(t) + MgR \sin(-\theta_{cr} - \theta(t)) = -M \ddot{u}_g(t) R \cos(-\theta_{cr} - \theta(t)), \quad \theta(t) < 0 \quad (1)$$

and

$$I_o \ddot{\theta}(t) + MgR \sin(\theta_{cr} - \theta(t)) = -M \ddot{u}_g(t) R \cos(\theta_{cr} - \theta(t)), \quad \theta(t) > 0 \quad (2)$$

where  $I_o$  is the mass moment of inertia about the centre of rotation,  $\ddot{\theta}$  is the angular acceleration,  $M$  is the mass of the block,  $g$  is gravitational acceleration,

$R$  ( $R = \sqrt{(h/2)^2 + (t/2)^2}$ ), is the

distance from the centre of mass to the pivot edge (see Fig. 2),  $\theta_{cr}$  is the critical overturning angle, and  $\theta$  is the rotation angle.

Eqs. 1 and 2 have been adopted in similar research (Yim *et al* 1980, Makris and Roussos 2000, among others) and are valid for arbitrary values of the critical angle  $\theta_{cr}$  ( $\theta_{cr} = \tan^{-1}(t/h)$ ). The frequency

parameter,  $p$ , for the rocking component is defined as  $p = \sqrt{MgR/I_o}$ ,  $I_o = (4/3)MR^2$  for rectangular blocks. Hence the frequency parameter can be simplified to  $\sqrt{3g/4R}$  for rectangular objects. It should be noted that the frequency parameter formula is different than the one predicted in the EDS modelling (refer to Fig. 1), which is a function only of the height of the component (i.e.  $p = \sqrt{3g/2H}$ ), in

which an assumption of  $R=h/2$  was adopted. This assumption limits the EDS model to very slender objects (e.g. walls). In the new RDS model the frequency parameter ( $p$ ) is applicable to any level of slenderness.

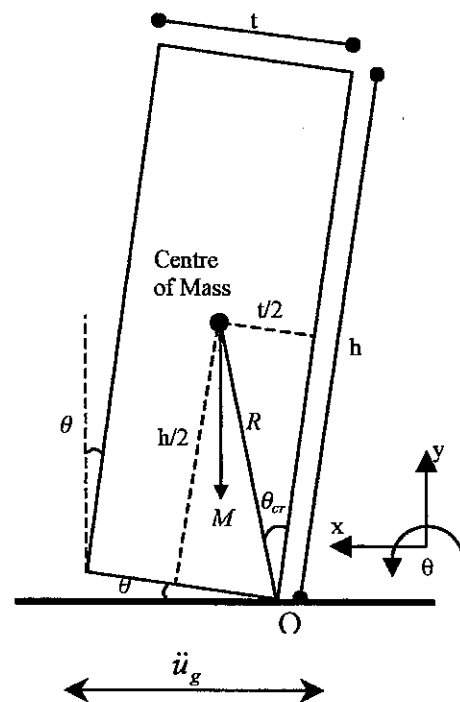


Fig. 2. Dimensions and parameters used to describe the rocking of a block.

Eqs.1 and 2 can be expressed in the compact form:

$$\ddot{\theta}(t) = -p^2 \{ \sin[\theta_{cr} \operatorname{sgn}[\theta(t)] - \theta(t)] + \frac{\ddot{u}_g}{g} \cos[\theta_{cr} \operatorname{sgn}[\theta(t)] - \theta(t)] \} \quad (3)$$

or

$$\ddot{\theta}(t) = -\frac{3}{4R} \{ g \sin[\theta_{cr} \operatorname{sgn}[\theta(t)] - \theta(t)] + \ddot{u}_g \cos[\theta_{cr} \operatorname{sgn}[\theta(t)] - \theta(t)] \} \quad (4)$$

The solution of the nonlinear problem of Eq. 4, need to be adjusted by accounting for the energy loss at every ground impact during rocking. The conservation of angular momentum principle (Ferdinand *et al*, 2004) could be applied at the instant of impact for predicting the dissipation in energy in terms of coefficient of restitution ( $R_D$ ), as shown by Eq. 5.

$$R_D = \frac{\text{Angular velocity after impact}}{\text{Angular velocity before impact}} = \frac{\dot{\theta}_2}{\dot{\theta}_1} = \frac{\frac{2R^2 - t^2}{2} + \frac{R^2}{3}}{R^2 + \frac{R^2}{3}} \quad (5)$$

The numerical integration of Eq. 4 along with the condition expressed by Eq. 5 yields the rotation time-history of a block for a given value for  $R$ , block thickness  $t$ , and ground excitation.

The solution algorithm for these equations can be implemented using a program language such as Fortran or C. In this research, Matlab has been used.

To illustrate the form and use of the RDS, a single pulse is applied as the ground or floor excitation. The displacement pulse and corresponding acceleration pulse are shown in Figs. 3a and 3b, respectively. Fig. 3c shows the resulting RDS for a rectangular component with different thicknesses ( $t$ ) varying from 400mm to 1000mm (the vertical axis of Fig. 3c and 4, presents the effective displacement that is defined at two-third the height of the object).

It can be seen from Fig. 3c, RDS provides direct prediction of rocking response for a given object, using only two geometric parameters ( $R$  and  $t$ ). The RDS directly presents the critical range of thicknesses for overturning, as illustrated for objects with thickness of 400mm.

Fig. 4 shows RDS for rectangular objects excited by an earthquake scenario. Similarly, Fig. 4 shows directly whether component would overturn or would rock. For the rocking situation, the maximum displacement at the effective height can be obtained.

In Figs. 3c and 4 the Elastic Displacement Spectra (EDS) are shown, to highlight the difference in prediction using these two methods. These two figures shows that the EDS can be highly non-conservative in predicting displacement, and hence the risk of overturning. Fig. 3c indicates that RDS predicts maximum displacement that is about twice that predicted by EDS. This ratio is found associated with only single pulse excitation and will be different with earthquake excitation, as illustrated by the comparison between RDS and EDS shown in Fig. 4. In this paper, only one accelerograms has been used to illustrate the effects of earthquake excitations characteristics by multiple cycles of pulse. Investigation is continuing with the use of accelerograms, which account for the filtering effects of the building.

Significantly, the RDS presentation can directly predict the minimum thickness of a component that will not overturn under a given excitation. For example, components with thickness greater than 600mm would not overturn when subjected to a single pulse

excitation (refer to Fig. 3c), while for the earthquake excitation, components with thickness more than 800mm are safe from overturning (refer to Fig. 4).

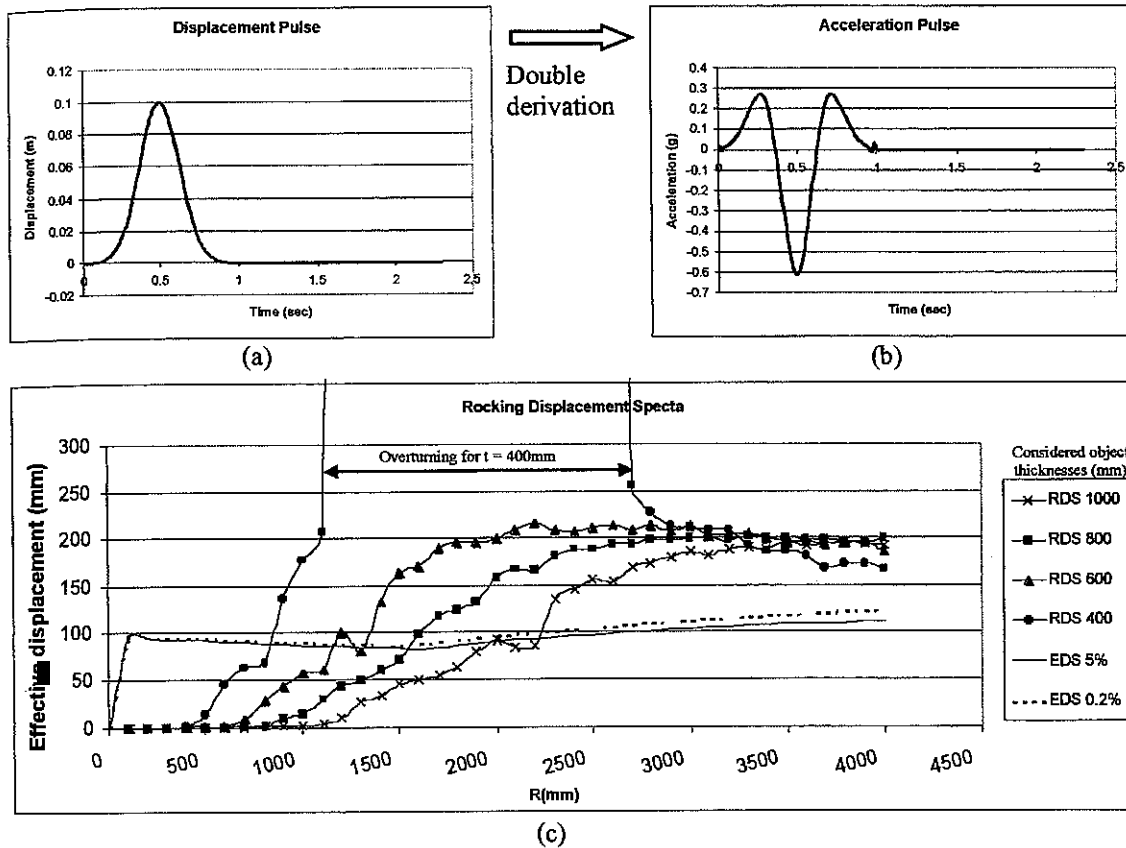


Fig. 3. Rocking Displacement Spectra (RDS) for a rectangular block with thicknesses of  $t = 400$  to  $1000\text{mm}$ , generated by single displacement pulse (shown below).

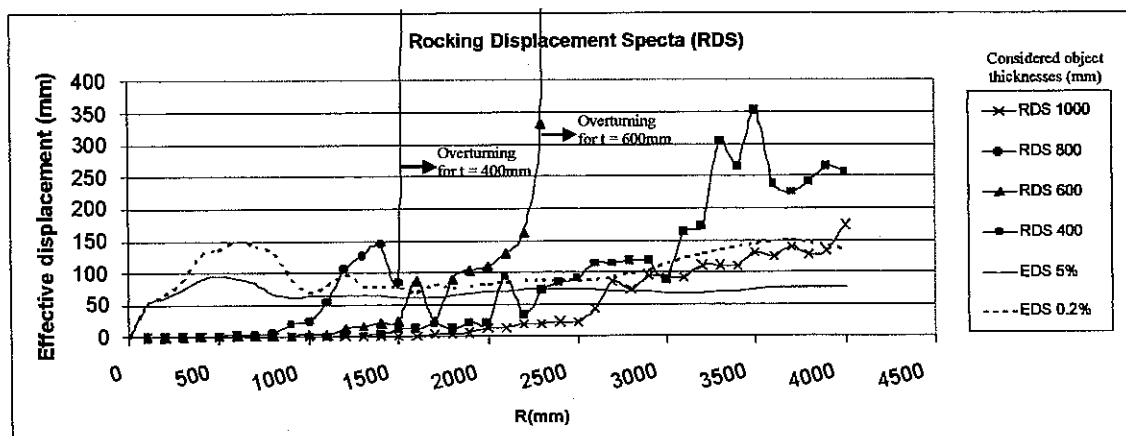


Fig. 4. Rocking Displacement Spectra (RDS) for a rectangular block with thicknesses of  $t = 400$  to  $1000\text{mm}$ , subjected to the 1940 El Centro (NS) California earthquake excitation.

#### 4. CONCLUSIONS

The modelling of the behaviour of unrestrained components subjected to earthquake (or explosion) excitation in buildings by a rational procedure has been considered in this

paper. A brief overview of the Elastic Displacement Spectra (EDS) approach for predicting the rocking response was presented and its limitations highlighted.

The new approach is performed by modelling the rocking problem in its real nonlinear form and also by applying the damping to the rocking motion at each impact during the rocking. With these procedures the exact Rocking Displacement Spectra (RDS) are predicted as a function of the component dimensions (i.e. thickness and height) as opposed to the object natural period in the EDS plot. Given a certain excitation on a building floor, RDS provides a direct indication of whether a component with specific dimensions will overturn or rock. For the rocking response case, RDS can show the maximum displacement expected during that excitation.

The use of RDS would enable direct and accurate assessment of free-standing equipment, particularly for the risk for overturning.

## 5. ACKNOWLEDGMENTS

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